Behavioral Sticky Prices

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Abstract

We develop a model in which households make decisions using a dual-process framework. System 1 relies on fast, intuitive heuristics but is prone to error, while System 2 demands cognitive effort but yields more accurate decisions. Monopolistic firms can influence which system households engage through pricing. This strategic influence creates a novel source of price inertia. The model accounts for the "rockets and feathers" phenomenon (prices rise quickly but fall slowly), explains why firms with unexpectedly high demand often avoid price changes, and why hazard functions are downward sloping. Our model implies that price stability is not optimal.

Keywords: E31, E32, E52, E71, Sticky prices, cognitive costs, System 1 and 2, rockets and feathers

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1. Introduction

We study a model in which households make decisions according to a dual-process framework widely used in the cognitive psychology literature to describe human decision making (see, e.g., Stanovich and West (2000)). In this framework, System 1 uses heuristics to make fast, low-effort decisions that are prone to errors. System 2 engages in slower, more deliberate reasoning that is cognitively costly but more accurate.

Our analysis builds on the elegant formulation of dual-process reasoning proposed by Ilut and Valchev (2023). This approach has two desirable features. First, individuals do not need to solve for the fully rational solution to assess how much to deviate from it, given cognitive or other costs. Second, individuals act more rationally when making high-stakes decisions than when facing less consequential ones.

In our model, there is a representative household that makes purchase errors because optimizing its consumption bundle involves cognitive effort. Since these errors are the same across households, we interpret them as corresponding to fads or fashions in consumption behavior. Monopolistic producers, for whom these errors result in high levels of demand relative to the rational optimum, have an incentive to keep their prices constant to discourage households from activating System 2 and reconsidering their purchasing decisions. This strategic behavior gives rise to a novel form of price inertia.

Our model is consistent with three important empirical facts. The first is the puzzling empirical regularity documented by Karrenbrock (1991), Neumark and Sharpe (1992), Borenstein, Cameron and Gilbert (1997), and Peltzman (2000) known as "rockets and feathers": prices rise rapidly when costs increase, but fall slowly when costs decrease. In our model, when costs rise substantially, all firms increase prices to avoid losses, leading costs and prices to rise in tandem. In contrast, when costs fall, firms enjoying strong demand have an incentive to keep prices fixed to avoid triggering reoptimization by consumers. As a result, prices decline, on average, more slowly than costs.

The second fact is the "sticky winners" phenomenon documented by Ilut, Valchev and Vincent (2020), whereby firms experiencing unexpectedly high demand at prevailing prices are less likely to adjust them. This behavior is central to our model: firms with favorable demand realizations avoid changing prices to prevent households from engaging System 2 and reoptimizing, which could trigger a new, potentially less favorable, demand shock.

The third empirical regularity is the downward-sloping hazard functions observed within narrowly defined goods categories (see Nakamura and Steinsson (2008) and Campbell and Eden (2014)). This pattern arises naturally from demand heterogeneity across firms producing the same type of good: those facing weak demand are more likely to adjust prices early, while firms experiencing strong demand tend to keep prices fixed for longer.

In standard models of cashless economies with sticky prices, price stability is typically optimal because it eliminates relative price distortions caused by inflation (see Woodford (2003)). In contrast, our framework implies that price stability is not optimal due to the strategic interaction between monopolistic firms and boundedly rational households. When average inflation is zero, there is still dispersion in the consumption of varieties because of cognitive errors. Firms that benefit from high demand keep their prices unchanged, which locks in consumption errors and leads households to settle on inefficient consumption bundles. Deflation is optimal because it increases the relative price of goods produced by sticky-price firms, reducing demand for those goods and mitigating the effects of behavioral biases.

We now present three empirical observations which, although not directly addressed by our model, provide suggestive evidence of the importance of System 1 in consumer behavior and of firms' strategic efforts to exploit this mode of decision making.

The first is "shrinkflation," a situation where manufacturers reduce product sizes while keeping prices constant. The UK Office for National Statistics (2019) identified 206 cases between September 2015 and June 2017 in which product size was reduced with prices remaining largely unchanged. Budianto (2024) reports that 35 percent of the products included in the U.K. consumer price index between 2012 and 2023 experienced changes in product size, with prices remaining constant in most instances. This practice suggests that some manufacturers are prepared to incur considerable expenses to keep prices stable, presumably to avoid triggering a re-optimization of household purchasing decisions.[2]

The second observation is the increasing adoption of subscription-based business models, such as streaming or software-as-a-service, and the tendency for subscription prices to remain stable over long periods. This stability can be interpreted as a tactic producers use to dissuade households from engaging System 2 and reassessing the value of their subscriptions.[3]

Amazon Prime subscription prices are remarkably sticky. Initially offered at an annual rate of $79 in 2011, the fee has only been adjusted a few times: to $99 in

[2]President Biden deemed shrinkflation important enough to merit discussion in a February 2024 Super Bowl video broadcast. The president noted that "sports drinks bottles are smaller, a bag of chips has fewer chips, but they're still charging us just as much [...] ice cream cartons have shrunk in size but not in price. [...] Some companies are trying to pull a fast one by shrinking the products little by little and hoping you won't notice."

[3]See Della Vigna and Malmendier (2006) for evidence that consumers often fail to rationally assess the value they derive from subscription services.

2014, $119 in 2018, and $139 in 2022. These adjustments were often accompanied by enhancements in service offerings, including the introduction of Amazon Prime Day, which served to justify the higher fees.

Netflix provides a case study of both price stability and shrinkflation. The standard subscription price remained at $7.99 from November 2010 until May 2014. At that point, the price was increased to $8.99, but only for new subscribers. Existing subscribers were grandfathered in at the $7.99 rate for an additional two years. Concurrently, Netflix rolled out a new basic plan priced at $7.99, which offered only standard-definition video on a single screen, a downgrade from the two high-definition screens available under the regular plan. The price for this basic plan remained unchanged until 2019.

The third observation is that convenient prices that are slightly below a round number (e.g., $9.99 instead of $10) are widely used (Kashyap (1995) and Blinder, Canetti, Lebow and Rudd (1998)), and less likely to change than other prices (Levy, Lee, Chen, Kauffman and Bergen (2011) and Ater and Gerlitz (2017)). This practice can be interpreted as a way to exploit System 1 thinking, creating the perception that the price is lower than its actual value.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 presents a version of the model with fully rational households, and Section 4 introduces bounded rationality into household decision-making. Section 5 shows that the model is consistent with the rockets and feathers phenomenon. Section 6 analyzes optimal fiscal and monetary policy. Section 7 develops a dynamic partial-equilibrium model of the firm and shows that it implies downward sloping hazard functions. Section 8 concludes.

2. Related literature

Our paper builds on the cognitive psychology literature (e.g., Evans and Stanovich (2013) and Stanovich and West (2000)), which distinguishes between two modes of decision-making: low-cost, heuristic thinking (System 1) and high-cost, analytical reasoning (System 2).

Ilut and Valchev (2023) develop a formulation of the dual-system framework and use it to study the household consumption-savings behavior in an incomplete markets environment. In familiar contexts, where beliefs about the policy function are precise, households rely on prior beliefs to make decisions. In unfamiliar situations, where beliefs are imprecise, households draw costly signals to update their beliefs about the policy function.

Building on Ilut and Valchev (2023), we model household decisions regarding the consumption of differentiated products. We show how strategic interactions between firms and boundedly rational consumers give rise to a new form of price inertia.

We extend Ilut and Valchev (2023)'s framework in two directions. First, we use a quadratic approximation to embed the tracking problem that determines signal precision within the utility maximization problem, rather than treating it separately. Second, we ensure that behavioral decisions satisfy the budget constraint directly, removing the need to specify a residual variable (the savings rate in their analysis) that adjusts so that the budget constraint holds.

The cognitive costs in our model are consistent with the findings of Afrouzi, Dietrich, Myrseth, Priftis and Schoenle (2024). Using survey evidence, these authors show that households prefer inflation to be zero. Seen through the lens of our model, this preference reflects the fact that cognitive costs are minimized when inflation is zero.

Our paper is linked to the literature on limited attention, limited information, or costly control by firms, including Mankiw and Reis (2002), Woodford (2009), Maćkowiak and Wiederholt (2009), Costain, Nakov and Petit (2019), and Ilut, Valchev and Vincent (2020).

It is also related to early work on near-rational behavior (e.g., Akerlof and Yellen (1985) and Cochrane (1989)) and recent work on behavioral macroeconomics, including Gabaix (2020), Angeletos and Lian (2023), Gabaix and Graeber (2024), Eichenbaum, Guerreiro and Obradovic (2024), and Andre, Flynn, Nikolakoudis and Sastry (2025).

In addition, our work relates to prior research on the strategic interaction between firms and consumers in models with information frictions. Matějka (2015) shows that firms strategically adopt a limited set of reference prices in the presence of inattentive consumers. De Clippel, Eliaz and Rozen (2014) explore how limited household attention impacts competition. Rotemberg (1982) proposes a framework where consumer anger over price changes incentivizes firms to limit price adjustments.

The mechanism in our model complements those that produce asymmetric price adjustments in menu cost models (see, e.g., Ellingsen, Friberg and Hassler (2006) and Burstein and Hellwig (2007)). Using a New Keynesian model with menu costs, Cavallo, Lippi and Miyahara (2023) show that prices tend to rise faster than they fall following significant cost shocks, such as the 2022 surge in energy prices. This phenomenon occurs because firms adjust prices more frequently when profit margins are under pressure. In order for these models to generate substantial price asymmetries, menu costs must be relatively large—around one percent of revenue (see Ellingsen, Friberg and Hassler (2006)).[4]

[4]According to data compiled by Aswath Damodaran (see data on operating and net margins by industry sector for the U.S. at this link) in January 2025, the pre-tax operating margin for grocery and food retail – defined as operating income (revenue minus cost of goods sold minus operating expenses) as a fraction of revenue – is 3.3 percent. So, a seemingly modest one percent menu cost would represent

An extended version of our model could potentially shed light on micro-level price rigidities that traditional models struggle to explain (see, for example, Alvarez, Le Bihan and Lippi (2014)). These phenomena include the presence of small price changes (Klenow and Kryvtsov (2008) and Eichenbaum, Jaimovich, Rebelo and Smith (2014)), the coexistence of high-frequency price changes with sticky reference prices (Eichenbaum, Jaimovich and Rebelo (2011)), and the observation that price adjustments for new products are larger and more frequent (Argente and Yeh (2022)).

At the macro level, our mechanism offers insights into the non-neutrality of monetary policy. Unlike standard menu-cost models (Golosov and Lucas (2007)), where firms with large price gaps dominate adjustments, our framework allows for heterogeneous endogenous adjustment costs. Firms with small price gaps may still adjust prices. As a result, monetary policy might be more effective.

Finally, our analysis complements other explanations of the rockets and feathers phenomenon. For instance, Tappata (2009) proposes a model where persistent cost shocks interact with consumers' limited information about market prices and production costs.

3. Model with fully rational households

In this section, we present a version of the model in which households face no cognitive costs when making decisions, and thus behave fully rationally. We describe the household problem, the problem of monopolistic producers, the government's fiscal and monetary policies, and the economy's equilibrium.

To streamline the presentation, we relegate the proofs of most lemmas and propositions in the remainder of the text to the Appendix.

3.1. Household problem

There is a representative household that maximizes its utility,

$$U = \frac{C^{1-\sigma} - 1}{1-\sigma} - \vartheta \frac{N^{1+\psi}}{1+\psi}, \quad \sigma, \psi, \vartheta > 0, \tag{1}$$

which depends on aggregate consumption (C) and hours worked (N).

Aggregate consumption results from a composite of differentiated goods,

$$C = \left(\int_0^1 C_i^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \; \theta > 1, \tag{2}$$

an implausibly large fraction – roughly 1/3 – of operating income.

where C_i denotes consumption of good i.

The household's budget constraint is given by,

$$\int_0^1 P_i C_i di \leq WN + \Pi - T, \tag{3}$$

where P_i is the nominal price of good i, W is the nominal wage, Π are total firm profits, and T are nominal taxes.

We solve the household problem in two steps.

Step 1. For a given level of consumption expenditure, E, determine the purchases of differentiated goods, C_i, that maximize the utility derived from consumption. The Lagrangian for this problem is

$$\mathcal{L}_e = \frac{C^{1-\sigma} - 1}{1-\sigma} + \Lambda_e \left(E - \int_0^1 P_i C_i di \right), \tag{4}$$

where C is given by equation (2).

The solution to this problem is given by,

$$C_i = \left(\frac{P_i}{P} \right)^{-\theta} \frac{E}{P}, \tag{5}$$

$$\Lambda_e = \left(\frac{E}{P} \right)^{-\sigma} \frac{1}{P}, \tag{6}$$

where

$$P \equiv \left(\int_0^1 P_i^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \tag{7}$$

Equations (2), (5), and (7) imply that $C = E/P$.

Step 2. Given the solutions for C_i as a function of E and of the prices of individual varieties, choose the optimal levels of total consumption expenditure and hours worked. Using the fact that $P\ C = E$, we can write the Lagrangian for this problem as,

$$\mathcal{L}_u = U(C, N) + \Lambda_u (WN + \Pi - T - PC). \tag{8}$$

The first-order conditions imply the familiar intratemporal condition for hours worked,

$$\vartheta C^{\sigma} N^{\psi} = \frac{W}{P}.$$

In the version of the model presented in Section 4, we focus on the implications of bounded rationality along a single dimension: the decision problem solved in step 1. We assume that decisions made in step 2 are fully rational.

3.2. Firm's Problem

Differentiated goods producers are monopolistically competitive. Firm i produces Y_i units of good i using N_i hours according to a linear production function,

$$Y_i = AN_i. \tag{9}$$

The firm's nominal profits, Π_i, are given by

$$\Pi_i = \left[P_i - (1-\tau)\frac{W}{A}\right]\left(\frac{P_i}{P}\right)^{-\theta} C.$$

where τ is the rate at which the government subsidizes wages.

The profit-maximizing price takes the familiar form,

$$P_i = \left(\frac{\theta}{\theta-1}\right)(1-\tau)\frac{W}{A}, \tag{10}$$

which implies that all firms choose the same price.

3.3. Fiscal and monetary policy

For simplicity, we assume that the central bank uses monetary policy to target nominal expenditure,

$$M = \int_0^1 P_i\, C_i\, di. \tag{11}$$

The government finances the wage subsidies provided to firms at a rate τ through lump-sum taxes,

$$T = \tau WN. \tag{12}$$

3.4. Equilibrium

Suppose $A = \overline{A}$, $M = \overline{M}$, and $\tau = \overline{\tau}$, where

$$1 - \overline{\tau} = \frac{\theta - 1}{\theta}.$$

This value of the labor subsidy eliminates the monopoly distortion, so that the price equals marginal cost.

Let $\overline{C}$, $\overline{N}$, and $\overline{P}$ denote the equilibrium values of aggregate consumption, labor, and the price level associated with $\overline{A}$, $\overline{\tau}$, and $\overline{M}$.

The equations above imply that,

$$\overline{C} = \left(\frac{1}{\vartheta}\right)^{\frac{1}{\sigma+\psi}} \overline{A}^{\frac{1+\psi}{\sigma+\psi}},$$

$$\overline{N} = \left(\frac{1}{\vartheta}\right)^{\frac{1}{\sigma+\psi}} \overline{A}^{\frac{1-\sigma}{\sigma+\psi}},$$

and

$$\overline{P} = \frac{\overline{M}}{\overline{C}}.$$

Since each firm's price equals marginal cost, profits are equal to the labor subsidies received: $\overline{\Pi} = \overline{T}$, and $\overline{P}\ \overline{C} = \overline{W}\ \overline{N}$.

3.5. A Second-Order Approximation

To set the stage for the study of household decisions under bounded rationality, we consider a log-quadratic approximation to the household's problem around the rational baseline equilibrium associated with $\overline{A}$, $\overline{\tau}$, and $\overline{M}$.

Throughout, unless stated otherwise, we use lowercase variables to denote the logarithmic deviation of a variable from the rational baseline equilibrium, i.e., for any X, $x \equiv \ln\left(X/\overline{X}\right)$. Given a function $f(X)$, we define $df \equiv f(X) - f(\overline{X})$.

The following lemma presents quadratic approximations for the two Lagrangians used to solve the household problem.

Lemma 1. *Let* $\hat{U} \equiv dU/\overline{C}^{1-\sigma}$, $\hat{\mathcal{L}}_e \equiv d\mathcal{L}_e/\overline{C}^{1-\sigma}$, $\hat{\mathcal{L}}_u \equiv d\mathcal{L}_u/\overline{C}^{1-\sigma}$. *Then*

$$\hat{U} \approx \int_0^1 c_i di + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\int_0^1 c_i^2 di + \frac{1}{2}\left[(1-\sigma) - \left(\frac{\theta-1}{\theta}\right)\right]\left(\int_0^1 c_i di\right)^2 - n - \frac{1}{2}(1+\psi)n^2,$$

$$\hat{\mathcal{L}}_e = -\frac{1}{2}\sigma c^2 - \frac{1}{2\theta}\mathrm{Var}_i(c_i) - \int_0^1 p_i c_i di + \lambda_e\big(e - p - c\big) + \Omega_e, \tag{13}$$

and

$$\begin{aligned} \hat{\mathcal{L}}_u = &-\frac{1}{2}\sigma c^2 - \frac{1}{2\theta}\mathrm{Var}_i(c_i) - \frac{1}{2}\psi n^2 + w n - \int_0^1 p_i c_i di \\ &+ \lambda_u\left[w + n + \tfrac{1}{\theta}\left(\ln\frac{\Pi}{\overline{\Pi}} - \ln\frac{T}{\overline{T}}\right) - p - c\right] + \Omega_u, \end{aligned} \tag{14}$$

where

$$p \equiv \int_0^1 p_i di, \tag{15}$$

$c \equiv \int_0^1 c_i di$, $\mathrm{Var}_i(c_i) \equiv \int_0^1 (c_i - c)^2 di$, *and* Ω_e *and* Ω_u *collect terms that are exogenous to the household problem.*

Under full rationality, the first-order conditions from Lagrangian (13) yield the standard demand function, which in logarithmic form is given by:

$$c_i^*\left(p_i\right) = e - p - \theta\left(p_i - p\right).$$

To set the stage for our discussion of bounded rationality, it is useful to restate the problem of optimally choosing consumption varieties as follows.

Lemma 2. *Let $\hat{\mathcal{L}}_e^*$ denote the Lagrangian (13) evaluated at the optimal values of c_i^* and λ_e^*. Define*

$$\Delta\hat{\mathcal{L}}_e \;\equiv\; \hat{\mathcal{L}}_e \;-\; \hat{\mathcal{L}}_e^*$$

as the deviation of $\hat{\mathcal{L}}_e$, the Lagrangian evaluated at arbitrary values, c_i and λ_e, from $\hat{\mathcal{L}}_e^*$. Then

$$\begin{aligned} \Delta\hat{\mathcal{L}}_e = -\frac{1}{2\theta}\Big[\int_0^1 \big[c_i - c_i^*(p_i)\big]^2 di \;+\; (\theta\,\sigma - 1)\Big(\int_0^1 \big[c_i - c_i^*(p_i)\big]\, di\Big)^2\Big] \\ + (\lambda_e - \lambda_e^*)\left(e - p - \int_0^1 c_i\, di\right), \end{aligned} \tag{16}$$

The proof of this lemma follows directly from the properties of quadratic forms. Under full rationality, the household chooses $\{c_i\}_{i\in[0,1]}$ and λ_e to maximize expression (16), which yields $c_i = c_i^*\left(p_i\right)$, $i \in [0, 1]$.

4. Model with boundedly-rational households

This section presents a version of the model in which households make decisions under bounded rationality along the lines of Ilut and Valchev (2023). We use this model to perform comparative statics with respect to different levels of aggregate productivity, A.

The model consists of two periods. In the pre-period, prior beliefs have high variance, so households rely on System 2 to choose c_i. In period one, households decide whether to use System 2 or rely on System 1, that is, use the decision rules established in the pre-period.

For System 1 to be well-defined in a static framework, we assume there is a pre-period in which prices are exogenously given initial conditions. These prices are taken as given by households in the pre-period and serve as state variables for firms and households in the main period.

To isolate the effects of bounded rationality on the choice of differentiated consumption goods, we assume that households make fully rational decisions about aggregate consumption and labor supply, as detailed in step 2 of Section 3. Bounded rationality applies only to the allocation of expenditure E across individual goods c_i, as described in step 1.

In our model, the optimal value of c_i has a simple form, which might lead a skeptical reader to ask why households do not simply compute the optimum. Our objective is not to argue that optimization is difficult in this stylized environment. Instead, we use this familiar monopolistic competition framework to illustrate how bounded rationality shapes consumer behavior and generates strategic interactions between firms and consumers.

Households can compute the demand for the baseline equilibrium associated with $\overline{A}$, $\overline{\tau}$, and $\overline{M}$, but are uncertain about how to respond to shocks. The household solves its problem using the second-order approximations described in Lemma 2.

The household enters the period with a prior belief $c_i^b(p_i)$, about $c_i^*(p_i)$ which follows a normal distribution,

$$c_i^b\left(p_i\right) \sim \mathcal{N}\left(\mu_i\left(p_i\right), \gamma_i^2\left(p_i\right)\right).$$

These distributions are independent across goods and values of p_i.

This formulation implies that cognitive uncertainty about the demand for good i depends only on its nominal price. Households know how to adjust the consumption of each variety i to changes in the aggregate price level, p, or aggregate consumption, c, but not in response to shifts in individual prices, p_i.

Households can obtain a noisy signal about the optimal consumption of variety i,

$$s_i(p_i) = c_i^*(p_i) + \gamma_\epsilon(p_i)\epsilon_i,$$

where $\epsilon_i \sim \mathcal{N}(0,1)$, and ϵ_i and ϵ_j are orthogonal for $i \neq j$.

This signal induces a posterior distribution for the optimal consumption of variety i, given by

$$c_i^b\left(p_i\right) \mid s_i \sim \mathcal{N}\left(\mu_{i|s}\left(p_i\right), \gamma_{i|s}^2\left(p_i\right)\right),$$

where $\mu_{i|s}\left(p_i\right)$ and $\gamma_{i|s}^2\left(p_i\right)$ are computed using the standard expressions for the conditional mean and variance of a normal distribution.

To generate a signal for the optimal consumption of good i, the household incurs a cognitive cost that increases with the precision of the signal. The utility of the boundedly rational household is $\hat{U} - \mathcal{I}$, where $\mathcal{I}$ is the total cognitive cost of all the signals generated by the household.

We assume that cognitive costs are proportional to the reduction in uncertainty. Following Sims (2003), we measure this reduction as the decrease in entropy, or equivalently, as the Shannon mutual information,

$$\mathcal{I} = \frac{\kappa}{2}\int_0^1 \left[\ln \gamma_i^2\left(p_i\right) - \ln \gamma_{i|s}^2\left(p_i\right)\right] di,$$

where

$$\gamma_i^2(p_i) \equiv \text{Var}\left[c_i^b(p_i)\right], \quad \gamma_{i|s}^2(p_i) \equiv \text{Var}\left[c_i^b(p_i) \mid s_i\right].$$

The expression for the conditional distribution of a normal random variable implies that

$$\gamma_{i|s}^2(p_i) = \frac{\gamma_i^2(p_i)\,\gamma_\epsilon^2(p_i)}{\gamma_i^2(p_i) + \gamma_\epsilon^2(p_i)},$$

so we can model the selection of the signal variance as a choice over the posterior variance, $\gamma_{i|s}^2(p_i)$.

It is helpful to define the expression analogous to (16) for the household making decisions under bounded rationality:[5]

$$\begin{aligned}\Delta\hat{\mathcal{L}}_e^b = &-\frac{1}{2\theta}\Big[\int_0^1 \left[c_i - c_i^b(p_i)\right]^2 di + (\theta\,\sigma - 1)\Big(\int_0^1 \left[c_i - c_i^b(p_i)\right] di\Big)^2\Big] \\ &+ \left(\lambda_e - \lambda_e^b\right)\left(e - p - \int_0^1 c_i\, di\right).\end{aligned} \tag{17}$$

Under bounded rationality, the problem of allocating spending across differentiated goods to maximize utility for a given total consumption expenditure can be written as

$$\max_{c_i, \gamma_{i|s}^2(p_i)} \min_{\lambda_e} \mathbb{E}\left(\Delta\hat{\mathcal{L}}_e^b\right) - \mathcal{I} \;\text{ s.t. }\; \gamma_{i|s}^2(p_i) \leq \gamma_i^2(p_i) \quad i \in [0,1], \tag{18}$$

where the constraint guarantees that the solution is consistent with Bayes' rule, that is, households cannot choose signals that increase the posterior variance to obtain a negative cognitive cost.[6]

We begin by solving for c_i. Using the fact that at any solution, $c = e - p$, the first-order conditions yield:

$$c_i = \mu_{i|s}(p_i) + c - \int_0^1 \mu_{i|s}(p_i)\, di. \tag{19}$$

[5]A well-known technical difficulty arises when computing the integral of independent, non-degenerate random variables indexed over an uncountable set: the process cannot be jointly measurable, and expressions such as $\int_0^1 X_i\, di$ are not well-defined as Riemann or Lebesgue integrals on a standard probability space (see Judd (1985) and Uhlig (1996)). We sidestep this measurability problem by interpreting the integral as the limit of an average over a large but finite set, $\int_0^1 X_i\, di \approx \frac{1}{N}\sum_{i=1}^N X_i$, which converges by the law of large numbers as $N \to \infty$.

[6]$\mathcal{I}$ does not include include cognitive costs associated with the Lagrange multiplier because these costs are not relevant to the solutions for c_i and $\gamma_{i|s}^2(p_i)$.

The demand for each good equals its posterior mean, adjusted by a constant term $(c - \int_0^1 \mu_{i|s}(p_i)\, di)$ to ensure that the constraint, $e - p = \int_0^1 c_i \, di$, is satisfied.

Having derived the demand functions given a set of signals, we now solve for the optimal posterior variance for each consumption variety, which is equivalent to selecting the optimal signal precision.

Lemma 3. *The optimal posterior variance of c_i^b is the solution to the following problem:*

$$\begin{aligned} &\max_{\gamma^2_{i|s}(p_i)} \; - \frac{1}{2\theta} \int_0^1 \gamma^2_{i|s}(p_i)\, di \; - \; \frac{\kappa}{2} \int_0^1 \Big[\ln\big(\gamma_i^2(p_i)\big) - \ln\big(\gamma^2_{i|s}(p_i)\big) \Big]\, di \\ &s.t. \quad \gamma^2_{i|s}(p_i) \; \leq \; \gamma_i^2(p_i), \quad i \in [0,1]. \end{aligned} \tag{20}$$

The first term in equation (20) captures the benefit from reducing uncertainty and the second term reflects the cognitive cost associated with achieving this reduction.

The first-order conditions for problem (20) are:

$$\gamma^2_{i|s}(p_i) = \min \left\{ \gamma_i^2(p_i) ; \theta\kappa \right\}. \tag{21}$$

This condition implies that the household activates System 2 for good i whenever the value of p_i is unfamiliar, i.e., when the prior uncertainty about the optimal value for c_i corresponding to p_i is high $(\gamma_i^2(p_i) > \theta\kappa)$.

The likelihood of activating System 2 declines with κ and θ. A higher κ increases cognitive costs, reducing the incentive to engage System 2. A higher θ implies greater substitutability across goods, reducing the utility contribution of each variety. Consequently, for a given level of aggregate consumption, c, the value of learning the optimal demand for each variety declines.

Learning in the pre-period. There is a pre-period in which households choose their consumption for each variety, i.

To ensure that ex-ante biases do not drive our results, we assume, as in Ilut and Valchev (2023), that the pre-period prior distribution is centered on the rational demand,

$$\mu_{i,0}(p_i) = c_i^*(p_i). \tag{22}$$

Obviously, households are not aware that their prior beliefs happen to be centered on the truth. If they were, they would already know the optimal choice and have no reason to expend cognitive effort.[7]

[7] An alternative way to eliminate systematic ex-ante biases is to assume that the prior mean equals the optimal demand plus a noise term uncorrelated across goods. In this case, the average ex-ante bias across goods is zero, but the prior for any individual good would generally not be centered on the true demand.

We assume that the pre-period prior variance, $\gamma_{i,0}^2(p_i)$, is equal to γ_c^2 for all goods. In addition, we assume that $\gamma_c^2 > \theta\kappa$, so the initial level of uncertainty justifies activating System 2.

Equation (21) implies that the household chooses to learn whenever its prior variance about optimal consumption of good i exceeds $\theta\kappa$. In the pre-period, all prior variances are, by assumption, above this threshold. As a result, the household generates a signal about the optimal consumption of good i corresponding to the prices set by firms, $p_{i,0}$, and updates its beliefs about this demand.

The household does not update its beliefs for prices not posted by firms in the pre-period. For those prices, the posterior distribution about optimal consumption is equal to the prior (recall that we assume the priors are centered on the rational demand). Given these considerations, the resulting pre-period posterior means and variances are:

$$\mu_i(p_i) = \begin{cases} c - \theta(p_i - p) + \alpha\gamma_\epsilon\epsilon_{i,0}, & \text{if } p_i = p_{i,0} \\ c - \theta(p_i - p), & \text{if } p_i \neq p_{i,0} \end{cases},$$

where

$$\alpha \equiv 1 - \frac{\theta\kappa}{\gamma_c^2}, \quad \gamma_\epsilon = \sqrt{\frac{\theta\kappa}{\alpha}}, \tag{23}$$

and

$$\gamma_i^2(p_i) = \begin{cases} \theta\kappa, & \text{if } p_i = p_{i,0} \\ \gamma_c^2, & \text{if } p_i \neq p_{i,0} \end{cases}.$$

Since $\gamma_c^2 > \theta\kappa$, the household relies on System 2 only when $p_i \neq p_{i,0}$. When the price of good i is the same as in the pre-period, uncertainty about the optimal consumption of good i is sufficiently low that the household chooses to avoid cognitive costs and follows the rule inherited from the pre-period.

The period-one posterior means, $\mu_{i|s}(p_i)$, are

$$\mu_{i|s}(p_i) = c - \theta(p_i - p) + \alpha\gamma_\epsilon\widetilde{\epsilon}_i, \tag{24}$$

where $\widetilde{\epsilon}_i = \epsilon_{i,0}$ if $p_i = p_{i,0}$, and $\widetilde{\epsilon}_i = \epsilon_{i,1} \sim \mathcal{N}(0,1)$ otherwise.

When firm i sets its price, it knows the value of $\epsilon_{i,0}$ but not $\epsilon_{i,1}$. By combining equations (19) and (24) we obtain the following expression for the demand for good i:

$$c_i = c - \theta(p_i - p) + \alpha\gamma_\epsilon\left[\widetilde{\epsilon}_i - \mathbb{E}_i(\widetilde{\epsilon})\right], \tag{25}$$

where the term $\mathbb{E}_i(\widetilde{\epsilon}) \equiv \int_0^1 \widetilde{\epsilon}_i di$ ensures that the constraint $e - p = \int_0^1 c_i\, di$ is satisfied.

Combining equations (14) and (25) and taking the first-order conditions with respect to c, n, and λ_u yields the standard intratemporal condition for labor choice, expressed in logarithmic form,

$$\sigma c + \psi n = w - p. \tag{26}$$

Our model has a representative household, so the demand shocks $\widetilde{\epsilon}_i$ are aggregate in nature. We interpret these shocks as capturing fads or fashions. Idiosyncratic demand shocks would have no effect, because they average out across households.

4.1. The firm's problem

We now revisit the firm's problem, taking into account the fact that households make decisions under bounded rationality. We write the problem in levels, using the relation $X = \overline{X}e^x$.

The ex-post nominal profits of firm i are given by,

$$\Pi_i = \left[P_i - (1-\tau)\frac{W}{A}\right] C_i.$$

The firm makes two decisions: whether to adjust its price, and if so, by how much.

Suppose that prices in the pre-period are common across firms, so $P_{i,0} = P_0$ for all $i \in [0,1]$. The following lemma characterizes the optimal pricing policy of firm i.

Lemma 4. *Define $\pi \equiv p - p_0$ and $\Theta \equiv \ln(\theta/(\theta-1))$. Firm i's optimal pricing policy is:*

$$p_i = \begin{cases} p_{adj}, & \text{if } \epsilon_{i,0} < \overline{\epsilon} \\ p_0, & \text{if } \epsilon_{i,0} \geq \overline{\epsilon} \end{cases}. \tag{27}$$

where

$$p_{adj} = w + \ln\left(\frac{1-\tau}{1-\overline{\tau}}\right) - a, \tag{28}$$

and

$$\overline{\epsilon} = \begin{cases} \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left\{(\theta-1)\left[(p_{adj}-p)+\pi\right] + \ln\left[\frac{1-e^{\left(p_{adj}-p\right)+\pi-\Theta}}{1-e^{-\Theta}}\right]\right\}, & \text{if } (p_{adj}-p)+\pi < \Theta \\ \infty, & \text{if } (p_{adj}-p)+\pi \geq \Theta \end{cases}. \tag{29}$$

Equation (27) implies that firms with a high demand shock (higher than $\overline{\epsilon}$) do not adjust their price. This behavior is consistent with the "sticky winners" phenomenon documented Ilut et al. (2020): firms experiencing unexpectedly high demand at prevailing prices are less likely to adjust them.

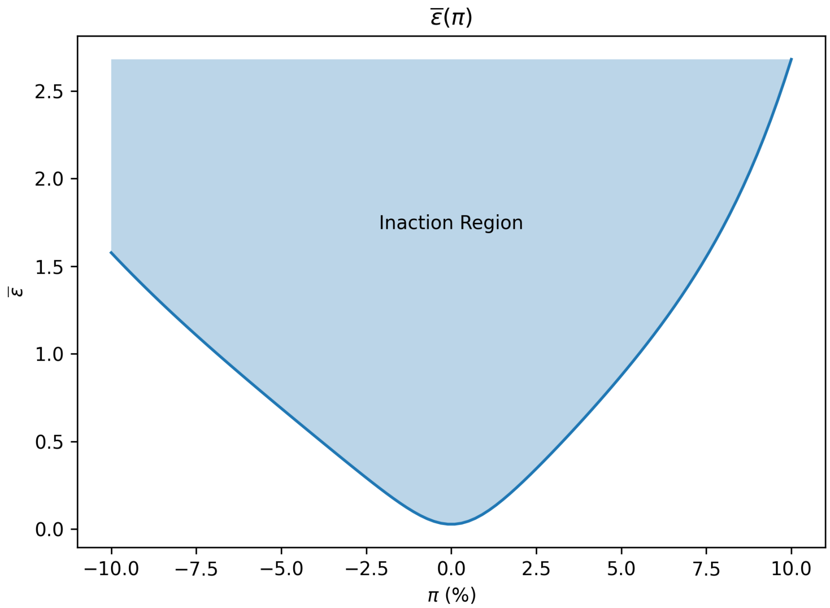

Figure 1: Inaction region for firms price setting

The optimal reset price, p_{adj}, coincides with the price in the model with fully rational households because, conditional on a price change, cognitive errors are uncorrelated with prices.

Equation (29) implies that when $(p_{\text{adj}} - p) + \pi \geq \Theta$, no realization of the past noise $\epsilon_{i,0}$ makes it optimal for the firm to keep its current price. In this case, the implied profit margin is non-positive, so the firm can always increase profits by adjusting its price. This asymmetry, which plays a central role in our "rockets and feathers" result, is illustrated in Figure 1.

High cost inflation erodes margins evaluated at fixed prices, prompting most firms to abandon their existing demand shocks and raise prices. In contrast, when costs decline, firms may choose to maintain their favorable demand shocks rather than reduce prices. Although a price cut could increase the quantity sold, some firms already sell more than they would if households were fully rational. Lowering prices would activate the household's System 2, prompting a reassessment of demand and generating a new demand shock. To avoid this reset and preserve their high demand levels, firms often choose to keep their prices constant.

The pricing policy described in Lemma 4 implies that a fraction χ of firms choose not to adjust their prices, where

$$\chi \equiv 1 - \Phi\left(\bar{\epsilon}\right), \tag{30}$$

and Φ is the cumulative distribution function of the standard normal distribution. Using the definition of the aggregate price level from equation (15), we obtain the following standard relationship between p_{adj} and π:

$$0 = -\chi\pi + (1-\chi)(p_{\text{adj}} - p). \tag{31}$$

4.2. Equilibrium

To define the equilibrium, we normalize the initial price level to one, which implies that the log price level at time zero is zero, $p_0 = 0$.

An equilibrium consists of allocations $\{c_i\}_{i\in[0,1]}, c, n$, prices $\{p_i\}_{i\in[0,1]}, w$, and information acquisition strategies $\{\gamma_{i|s}\}_{i\in[0,1]}$, such that, given a, m, τ, p_0, and $\{\mu_i, \gamma_i\}_{i\in[0,1]}$, the following conditions are satisfied:

1. Given e, the price vector $\{p_i\}_{i\in[0,1]}$, and the belief parameters $\{\mu_i, \gamma_i\}_{i\in[0,1]}$, the household chooses c_i and $\{\gamma_{i|s}\}_{i\in[0,1]}$ to solve the optimization problem (18);
2. Given consumption decisions for c_i, the household chooses c and n to maximize utility;
3. Each firm i chooses p_i to maximize profits;
4. The aggregate price level p satisfies equation (15);
5. Markets clear:

$$\pi + c = m, \tag{32}$$

$$\int_0^1 n_i \, di = n. \tag{33}$$

$$c_i = a + n_i \tag{34}$$

The government budget constraint is redundant.

Using equations (33) and (34) we obtain:

$$c = a + n, \tag{35}$$

which shows that, to a first-order approximation, there are no productive distortions.

The equilibrium conditions for the aggregate variables are given by equations (26), (28), (29), (30), (31), (32), and (35). Substituting equations (28) and (31) into this system of equations allows us to reduce the equilibrium conditions to equation (30), (32), and the condition for the cutoff $\bar{\epsilon}$:

$$\bar{\epsilon} = \begin{cases} \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[(\theta-1)\frac{\pi}{1-\chi} + \ln\left(\frac{1-e^{\frac{\pi}{1-\chi}-\Theta}}{1-e^{-\Theta}}\right)\right], & \text{if } \frac{\pi}{1-\chi} < \Theta, \\ \infty, & \text{if } \frac{\pi}{1-\chi} \geq \Theta, \end{cases}$$

and

$$c = c^* + \frac{1}{\psi+\sigma}\left[\frac{\chi}{1-\chi}\pi - \ln\left(\frac{1-\tau}{1-\bar{\tau}}\right)\right],$$

where

$$c^* \equiv \left(\frac{1+\psi}{\psi+\sigma}\right)a,$$

denotes aggregate output in the equilibrium of the model with fully rational households.

The following proposition summarizes the existence and uniqueness properties of our model.

Proposition 1. *An equilibrium exists. Moreover, if $\psi + \sigma \geq 1$, the equilibrium is unique.*

4.3. The Phillips curve

For $\tau = \bar{\tau}$, the Phillips curve for this economy is given by

$$c - c^* = \frac{1}{\psi+\sigma}\left(\frac{\chi}{1-\chi}\right)\pi. \tag{36}$$

Figure 2 displays this Phillips curve. The output gap is defined as the current level of log output minus the level of log output in the economy with fully rational households. When inflation exceeds 12 percent, the Phillips curve becomes approximately vertical: firms face low or negative profit margins at current prices and choose to adjust prices regardless of their demand shocks. The Phillips curve has a conventional upward slope when the inflation rate is between -3.8 and 3 percent. In this range, higher inflation is associated with a more positive output gap. As inflation rises beyond 3 percent, price flexibility increases as more firms adjust their prices, causing the Phillips curve to slope backward and the output gap to approach zero. Because firms are more responsive to inflation than to deflation, the inflation rate at which the Phillips curve becomes approximately vertical is lower in absolute value when inflation is positive (12 percent) than negative (19 percent).

We now examine the properties of the equilibrium. The following proposition characterizes the relationship between the threshold $\bar{\epsilon}$ and inflation π, using the fact that $\chi = 0$ when $\bar{\epsilon} = \infty$.

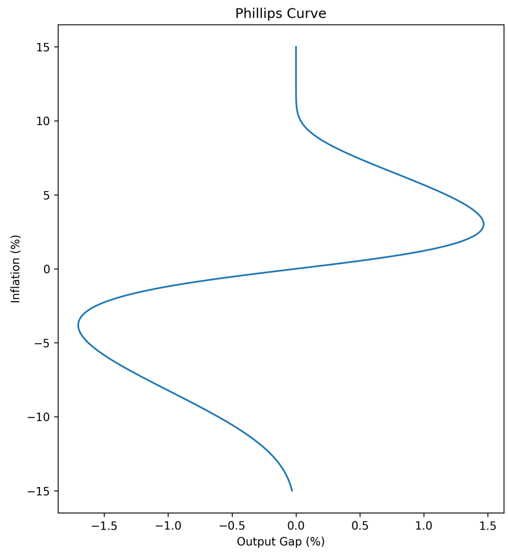

Figure 2: Phillips curve: inflation rate versus output gap.

Proposition 2. *The equilibrium relationship between $\bar{\epsilon}$ and π is given by:*

$$\bar{\epsilon}(\pi) = \begin{cases} \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[(\theta-1)\frac{\pi}{1-\chi} + \ln\left(\frac{1-e^{\frac{\pi}{1-\chi}-\Theta}}{1-e^{-\Theta}}\right)\right], & \text{if } \pi < \Theta, \\ \infty, & \text{if } \pi \geq \Theta. \end{cases} \quad (37)$$

Moreover, the function $\bar{\epsilon}(\pi)$ satisfies the following properties:

1. *$\bar{\epsilon}(\pi)$ attains its minimum at $\pi = 0$;*
2. *For any $a > 0$, we have $\bar{\epsilon}(a) > \bar{\epsilon}(-a)$.*

Proposition 2 implies that the function

$$\chi(\pi) \equiv 1 - \Phi\left[\bar{\epsilon}(\pi)\right],$$

attains its maximum at $\pi = 0$ and satisfies $\chi(a) < \chi(-a)$ for all $a > 0$. Hence, the model implies an asymmetry in the hazard expressed as a function of the price gap. The probability of a change in price rises faster with positive inflation than with deflation. This asymmetry plays a key role in explaining the rockets and feathers phenomenon discussed in Section 5.

5. Rockets and Feathers

We now study the impact of cost shocks and show that our model is consistent with the rockets and feathers phenomenon: prices rise quickly when costs increase but fall slowly when costs fall.

To do so, we examine the equilibrium response to symmetric cost shocks, $\nu > 0$ and $-\nu$, assuming $m = 0$. We define the deviation of consumption from its steady-state value as

$$\widetilde{c}(\pi) \equiv c(\pi) - c^* = \frac{1}{\psi + \sigma} \left(\frac{\chi}{1 - \chi} \right) \pi.$$

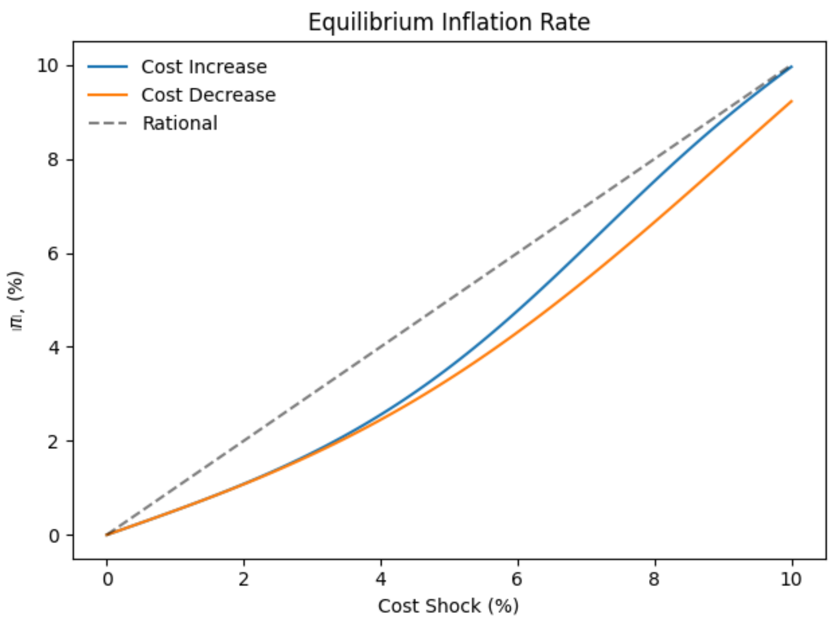

Figure 3: The impact of cost shocks on the absolute value of the logarithm of inflation

A cost increase ($\nu < 0$) leads to inflation, while a cost decrease ($\nu > 0$) results in deflation. To compare the price response to both types of shocks, Figure 3 plots the absolute value of the logarithm of gross inflation against the absolute value of the cost shock, $|\nu|$. The blue line represents cost increases, and the orange line cost decreases. In a fully rational version of our model, the two lines would coincide, because price responses would be symmetric. With bounded rationality, prices respond more to cost increases than to cost decreases.

This "rockets and feathers" pattern results from an underlying asymmetry in the profit function. Although the function is locally symmetric around the optimal price, it is globally asymmetric. When costs rise, profit margins eventually turn negative. At that point, firms cannot, as the old business adage goes, "lose money on every unit but make it up on volume." Cost increases ultimately prompt firms to abandon a favorable demand shock and reoptimize their price.

In contrast, when costs fall, profit margins improve rather than deteriorate. Firms benefiting from strong demand prefer to keep their prices unchanged to avoid triggering System 2 and generate a new demand shock. As a result, average price reductions are typically smaller than cost declines.

As $|\nu|$ increases, the orange and blue curves in Figure 3 converge. When positive cost shocks exceed roughly 12 percent, most firms adjust their prices. In contrast, it takes a cost decline of more than 19 percent for nearly all firms to reduce prices.

The following proposition states our key result:

Proposition 3. *Let $\nu > 0$, and consider the equilibria with $c^* = \nu$ and $m = 0$. Then the corresponding inflation rates satisfy $\pi(\nu) < 0$, $\pi(-\nu) > 0$, and*

$$\pi(-\nu) > -\pi(\nu).$$

For sufficiently large cost shocks, inflation responds more strongly (in percentage terms) than deflation to shocks of equal absolute size.

The asymmetry in the profit function also gives rise to asymmetric price adjustments in other frameworks, such as menu cost models (see Ellingsen, Friberg and Hassler (2006), Burstein and Hellwig (2007) and Cavallo, Lippi and Miyahara (2023)).

6. Optimal Policy

We now characterize the optimal values for the labor subsidy rate, τ, and the growth rate of money, m. We start by computing the indirect utility, net of cognitive costs, for a particular equilibrium.

Lemma 5. *The Lagrangian associated with the quadratic approximation to the planner's problem, $\hat{\mathcal{L}}_p \equiv d\mathcal{L}_p/\overline{C}^{1-\sigma}$, is*

$$\hat{\mathcal{L}}_p = -\frac{1}{2}\sigma c^2 - \frac{1}{2}\psi n^2 - \frac{1}{2\theta}\Delta\left(\pi\right) + a + \frac{1}{2}a^2 + an + \lambda_p\left(n + a - c\right).$$

where

$$\Delta\left(\pi\right) \equiv \mathrm{Var}_i\left(c_i\right) + 2\theta\mathcal{I},$$

$$\mathrm{Var}_i(\, c_i\,) = \theta^2\,\frac{1-\Phi\big(\bar{\epsilon}(\pi)\big)}{\Phi\big(\bar{\epsilon}(\pi)\big)}\,\pi^2 \;+\; \alpha^2\,\gamma_\epsilon^2\Big\{\,1 + \bar{\epsilon}\,\phi\big(\bar{\epsilon}(\pi)\big) \;-\; \phi^2\big(\bar{\epsilon}(\pi)\big)\Big\}$$
$$+\,2\,\theta\,\alpha\,\gamma_\epsilon\,\pi\,\phi\big(\bar{\epsilon}(\pi)\big)\,,$$

$$2\theta\mathcal{I} = \Phi\left[\bar{\epsilon}\left(\pi\right)\right]\alpha\gamma_{\epsilon}^{2}\ln\left(\frac{1}{1-\alpha}\right),$$

and λ_p *denotes the Lagrange multiplier.*

The solution to the planner's problem involves

$$c = c^{*} = \left(\frac{1+\psi}{\sigma+\psi}\right)a,$$

and

$$n = n^{*} = \left(\frac{1-\sigma}{\sigma+\psi}\right)a.$$

That is, the optimal allocation corresponds to the first-order responses in the rational economy.

Lemma 5 implies that the optimal inflation rate solves

$$\min_{\pi}\Delta\left(\pi\right).$$

We first show that if prior uncertainty is sufficiently high, then price stability ($\pi = 0$) is preferable to high inflation ($\pi \geq \Theta$).

Lemma 6. *There is* $\overline{\gamma}_c$ *such that, if* $\gamma_c \geq \overline{\gamma}_c$*, then* $\Delta\left(0\right) < \Delta\left(\Theta\right)$*.*

We now show that even under parameter conditions that ensure that price stability is preferable to full flexibility, it is not optimal to set $\pi = 0$.

Lemma 7. *There is* $\delta > 0$ *such that for all* $\pi \in (-\delta, 0)$*,* $\Delta\left(\pi\right) < \Delta\left(0\right)$*.*

Proof. The result follows from the fact that

$$\Delta'\left(0\right) = 2\theta\alpha\gamma_{\epsilon}\phi\left[\bar{\epsilon}\left(0\right)\right] > 0.$$

□

The intuition for this result is as follows. When average inflation is zero, firms experiencing high demand due to household decision errors do not change their prices. Other firms slightly increase or decrease their prices to draw a new demand shock. As a result, sizeable behavioral mistakes become ingrained, leading households to select a highly suboptimal consumption basket. Moving from zero inflation to deflation mitigates this inefficiency by improving consumption choices.

Why is deflation locally better than inflation? The logic is as follows. Due to cognitive costs, households do not choose the fully-rational value of c_i. The planner would like to reduce the consumption of goods supplied by firms that have sticky prices, since these firms received positive demand shocks. When inflation is positive, the relative price of the goods produced by firms with sticky prices falls, inducing households to consume more of these goods and exacerbating the impact of behavioral biases. In contrast, when inflation is negative, the relative price of the goods produced by firms with sticky prices rises. As a result, the consumption of these goods falls, mitigating the impact of behavioral biases.

7. A Dynamic Model

In this section, we consider a dynamic partial equilibrium model where a single firm faces the dual-process demand discussed in Section 4 and maximizes the expected present value of profits discounted at rate β. We now omit the subscript i and denote by P_t the price set by the firm. We assume that the aggregate price level is constant and normalized to one.

Households have incomplete memory: when the price changes, they forget past prices and purchase decisions, and learn a noisy estimate of their optimal demand at the new price. So, prices observed before period $t-1$ are irrelevant for household choices.

As in the static model, firms choose prices and then households choose whether to draw a signal about the optimal demand. The firm's demand is given by:

$$C_t = P_t^{-\theta} \begin{cases} e^{\alpha\gamma_\epsilon \epsilon_{t-1}}, & \text{if } p_t = p_{t-1} \\ e^{\alpha\gamma_\epsilon \epsilon_t}, & \text{if } p_t \neq p_{t-1} \end{cases},$$

where $\epsilon_t \sim \mathcal{N}(0,1)$. As before, lowercase variables denote logarithmic deviations, e.g. $\ln(P_t/1) = p_t$.

The marginal cost, Ξ_t, is observed at the beginning of the period before the firm makes its pricing decision. When the firm changes its price, the new demand shock is observed at the end of the period. Expected flow profits at the beginning of the period are given by:

$$\Pi_t = (P_t - \Xi_t)\, P_t^{-\theta} \begin{cases} e^{\alpha\gamma_\epsilon \epsilon_{t-1}}, & \text{if } p_t = p_{t-1} \\ \mathbb{E}\left(e^{\alpha\gamma_\epsilon \epsilon_t}\right), & \text{if } p_t \neq p_{t-1}, \end{cases}.$$

To simplify, we use a second-order log-approximation to flow profits around the following solution to the firm's problem: $\overline{\Xi} \equiv (\theta-1)/\theta$, $\overline{P} \equiv 1$, and $\overline{\Pi} = 1/\theta$.

We assume that the logarithm of marginal cost, $\xi = \ln\left(\Xi_t/\overline{\Xi}\right)$, evolves according to a jump diffusion process,

$$\xi_t = \xi_{t-1} + \upsilon_t,$$

where the innovation υ_t follows the following process:

$$\upsilon_t = \begin{cases} 0, & \text{with probability } \rho \\ \sim \mathcal{N}\left(0, \gamma_\upsilon^2\right), & \text{with probability } 1-\rho \end{cases}.$$

Lemma 8. *Let* $r_t \equiv \ln\left(\Pi_t/\overline{\Pi}\right)$*. The firm's per-period reward, computed using a second-order approximation, is*

$$r_t = -\left(\theta-1\right)\xi_t - \frac{\theta\left(\theta-1\right)}{2}\left(p_t-\xi_t\right)^2 + \begin{cases} \alpha\gamma_\epsilon\epsilon_{t-1}, & \text{if } p_t = p_{t-1} \\ \frac{1}{2}\left(\alpha\gamma_\epsilon\right)^2, & \text{if } p_t \neq p_{t-1} \end{cases}.$$

Let $x_t = p_{t-1} - \xi_t$ denote the beginning-of-period price gap, and let $\widetilde{x}_t = p_t - \xi_t$ be the price gap chosen in period t. The jump diffusion process for marginal cost, implies:

$$x_{t+1} = p_t - \xi_{t+1} = \widetilde{x}_t - \upsilon_{t+1}.$$

The firm's problem can be formulated recursively with two state variables: x_t and ϵ_{t-1}. Let $\beta < 1$ denote the firm's discount factor. The firm's value function is given by:

$$V\left(x,\epsilon\right) = \max\left\{V_{\text{no-adj}}\left(x,\epsilon\right); V_{\text{adj}}\right\}, \tag{38}$$

where

$$V_{\text{no-adj}}\left(x,\epsilon\right) = \left(1-\beta\right)\left[-\frac{\theta\left(\theta-1\right)}{2}x^2 + \alpha\gamma_\epsilon\epsilon\right] + \beta\mathbb{E}_\upsilon\left[V\left(x-\upsilon',\epsilon\right)\right], \tag{39}$$

and

$$V_{\text{adj}} = \max_{\widetilde{x}}\left\{\left(1-\beta\right)\left[-\frac{\theta\left(\theta-1\right)}{2}\widetilde{x}^2 + \frac{1}{2}\left(\alpha\gamma_\epsilon\right)^2\right] + \beta\mathbb{E}_\epsilon\left[\mathbb{E}_\upsilon\left[V\left(\widetilde{x}-\upsilon',\epsilon'\right)\right]\right]\right\}. \tag{40}$$

The following lemma describes some key properties of the firm's value function.

Lemma 9. $V_{\text{no-adj}}\left(x,\epsilon\right)$ *is strictly increasing in* ϵ *and* $V\left(x,\epsilon\right)$ *is nondecreasing in* ϵ*.*

Proof. Suppose $V\left(x,\epsilon\right)$ is nondecreasing in ϵ. From equation (39), $V_{\text{no-adj}}\left(x,\epsilon\right)$ is strictly increasing in ϵ. Since V_{adj} is a constant, the operator implied by equation (38) maps into a nondecreasing function. Because the space of nondecreasing functions is closed, $V\left(x,\epsilon\right)$ is nondecreasing. □

Corollary 1. *The optimal policy involves a threshold* $\overline{\epsilon}\left(x\right)$ *such that if* $\epsilon > \overline{\epsilon}\left(x\right)$*,* $V_{\text{no-adj}}\left(x,\epsilon\right) > V_{\text{adj}}$*.*

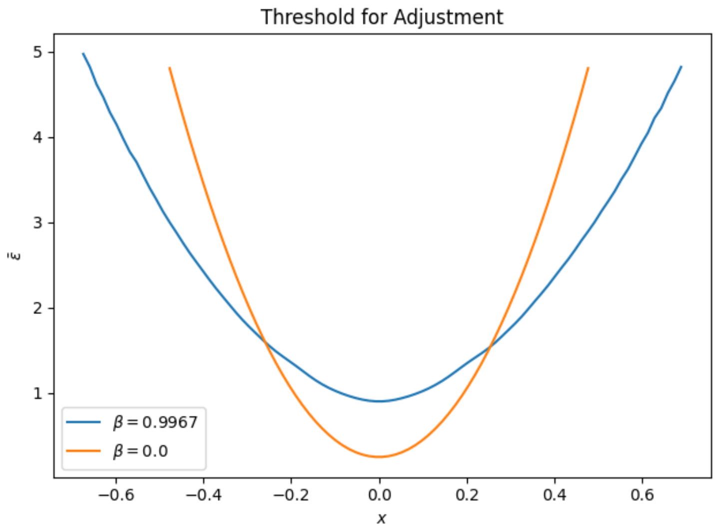

Figure 4: Inaction region

Figure 4 shows how the discount factor affects the inaction region—the set of conditions under which firms keep prices unchanged. The orange curve represents myopic firms ($\beta = 0$), while the blue curve corresponds to forward-looking firms that value future profits.[8]

Myopic firms place less value on favorable demand shocks than forward-looking firms. This property has two implications illustrated in Figure 4. First, when the price gap is small, myopic firms are less inclined to adjust prices to activate System 2 in hopes of eliciting a strong demand realization, because they fail to account for the future value of a high demand shock. Second, when the price gap is large, myopic firms require unusually strong demand shocks to justify leaving prices unchanged, again because they disregard the future benefits of high demand. In contrast, forward-looking firms are more likely to keep prices fixed even with large price gaps, because they recognize that positive demand shocks are valuable in the future.

We now highlight an important property of the dynamic model: it can account for a key empirical regularity emphasized by Nakamura and Steinsson (2008) and Campbell and Eden (2014): the hazard function for individual goods categories is downward

[8]Figure 4 is analogous to Figure 1. Although Figure 1 displays inflation on the x-axis, inflation effectively determines the price gap in the static model. However, Figure 4 lacks the asymmetries seen in Figure 1 because the firm problem is solved using a quadratic approximation.

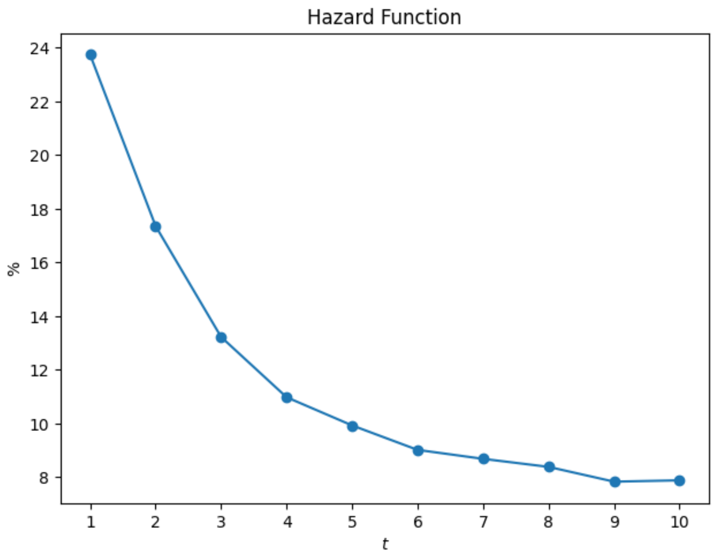

Figure 5: Hazard function: probability that a firm changes its price in period t, conditional on the price having remained unchanged for the previous $t-1$ periods

sloping.[9] In contrast, standard menu cost models typically generate upward-sloping hazard functions.

Figure 5 plots the hazard function implied by our model.[10] In our framework, firms facing unfavorable demand shocks are more likely to adjust prices early, while those experiencing favorable shocks tend to keep prices fixed for longer periods. This heterogeneity in demand conditions naturally gives rise to downward-sloping hazard rates.

[9]The aggregate hazard function across all CPI categories is sharply downward-sloping (see, e.g., Klenow and Kryvtsov (2008)). This property is primarily due to a composition effect across different categories. Prices of goods such as gasoline and fresh food products change frequently, while service prices are more stable. At short durations, all categories are represented, but at longer durations, services dominate. Our focus is not on these compositional effects across categories but on the fact that hazard functions tend to be downward-sloping even within narrowly defined categories.

[10]See Alvarez, Lippi and Oskolkov (2022) for a discussion of the close relation between the duration hazard depicted in Figure 4 and the "generalized hazard function" displayed in Figure 5, which relates the firm's price adjustment probability to its own state.

8. Conclusion

This paper develops a model in which households make decisions according to a dual-process framework. This approach gives rise to a novel form of price rigidity that stems from the strategic interaction between consumers and monopolistic producers. There is a range of cost shocks for which some producers refrain from adjusting prices so that households do not reassess their purchasing decisions.

The model is consistent with three important empirical facts. First, it accounts for the well-known "rockets and feathers" phenomenon: prices rise quickly in response to cost increases but fall slowly when costs decline. Second, it is consistent with the finding of Ilut et al. (2020) that firms experiencing strong demand realizations are less likely to adjust their prices. Third, it produces downward-sloping hazard functions, consistent with those estimated from micro data.

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9. Appendix

9.1. Proof of Lemma 1

Let

$$u(\mathbf{c}) \;\equiv\; \frac{\left(\frac{1}{n}\sum_{k=1}^{n} C_{\frac{k}{n}}^{\frac{\theta-1}{\theta}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}} - 1}{1-\sigma},$$

and let

$$C_{\frac{k}{n}} \;\equiv\; \overline{C}\, e^{c_{\frac{k}{n}}} .$$

The vector $\mathbf{c}$ is given by $\left[c_{\frac{1}{n}}, c_{\frac{2}{n}}, ...c_{\frac{n}{n}}\right]$. Then

$$u(\mathbf{c}) = \frac{\overline{C}^{\,1-\sigma}\left(\frac{1}{n}\sum_{k=1}^{n} e^{\frac{\theta-1}{\theta}\,c_{\frac{k}{n}}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}} \;-\; 1}{1-\sigma} .$$

The derivative of $u(\mathbf{c})$ with respect to $c_{\frac{k}{n}}$ is

$$u_k(\mathbf{c}) = \overline{C}^{\,1-\sigma}\left(\frac{1}{n}\sum_{i=1}^{n} e^{\frac{\theta-1}{\theta}\,c_{\frac{i}{n}}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}-1} \frac{1}{n}\, e^{\frac{\theta-1}{\theta}\,c_{\frac{k}{n}}} .$$

The derivative of $u_k(\mathbf{c})$ with respect to $c_{\frac{j}{n}}$ is

$$u_{kj}(\mathbf{c}) = \overline{C}^{\,1-\sigma}\left[(1-\sigma)\,\frac{\theta}{\theta-1} \;-\; 1\right]\left(\frac{1}{n}\sum_{i=1}^{n} e^{\frac{\theta-1}{\theta}\,c_{\frac{i}{n}}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}-2}$$
$$\times\left(\frac{1}{n}\, e^{\frac{\theta-1}{\theta}\,c_{\frac{k}{n}}}\right)\left(\frac{1}{n}\right)\left(\frac{\theta-1}{\theta}\right) e^{\frac{\theta-1}{\theta}\,c_{\frac{j}{n}}} .$$

The derivative of $u_k(\mathbf{c})$ with respect to $c_{\frac{k}{n}}$ is

$$u_{kk}(\mathbf{c}) = \overline{C}^{\,1-\sigma}\left[\left(\frac{1}{n}\, e^{\frac{\theta-1}{\theta}\,c_{\frac{k}{n}}}\right)\left[(1-\sigma)\,\frac{\theta}{\theta-1} \;-\; 1\right]\left(\frac{1}{n}\sum_{i=1}^{n} e^{\frac{\theta-1}{\theta}\,c_{\frac{i}{n}}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}-2}\right.$$
$$\left.\times\left(\frac{1}{n}\right)\left(\frac{\theta-1}{\theta}\right) e^{\frac{\theta-1}{\theta}\,c_{\frac{k}{n}}}\right]$$
$$+\,\overline{C}^{\,1-\sigma}\left(\frac{1}{n}\sum_{i=1}^{n} e^{\frac{\theta-1}{\theta}\,c_{\frac{i}{n}}}\right)^{(1-\sigma)\frac{\theta}{\theta-1}-1}\left(\frac{\theta-1}{\theta}\right)\left(\frac{1}{n}\, e^{\frac{\theta-1}{\theta}\,c_{\frac{k}{n}}}\right).$$

Evaluating these expressions at $\mathbf{c} = \mathbf{0}$, we obtain

$$u(\mathbf{0}) = \frac{\overline{C}^{\,1-\sigma} - 1}{1-\sigma}, \qquad u_k(\mathbf{0}) = \frac{1}{n}\,\overline{C}^{\,1-\sigma},$$

$$u_{kj}(\mathbf{0}) = \overline{C}^{\,1-\sigma}\left[(1-\sigma)\ -\ \left(\tfrac{\theta-1}{\theta}\right)\right]\left(\tfrac{1}{n}\right)^2,$$

and

$$u_{kk}(\mathbf{0}) = \overline{C}^{\,1-\sigma}\left[(1-\sigma)\ -\ \left(\tfrac{\theta-1}{\theta}\right)\right]\left(\tfrac{1}{n}\right)^2\ +\ \overline{C}^{\,1-\sigma}\left(\tfrac{\theta-1}{\theta}\right)\frac{1}{n}.$$

A quadratic approximation to $u(\mathbf{c})$ around $\mathbf{c} = 0$ is given by,

$$\begin{aligned} u(\mathbf{c}) \approx u(\mathbf{0})\ &+\ \overline{C}^{\,1-\sigma}\,\frac{1}{n}\sum_{k=1}^{n} c_{\frac{k}{n}} \\ &+ \frac{1}{2}\,\overline{C}^{\,1-\sigma}\left(\tfrac{\theta-1}{\theta}\right)\frac{1}{n}\sum_{k=1}^{n} c_{\frac{k}{n}}^2 \\ &+ \frac{1}{2}\,\overline{C}^{\,1-\sigma}\left[(1-\sigma)\ -\ \left(\tfrac{\theta-1}{\theta}\right)\right]\left(\tfrac{1}{n}\sum_{k=1}^{n} c_{\frac{k}{n}}\right)^2. \end{aligned}$$

Taking $n \to \infty$, we obtain,

$$\frac{u(\mathbf{c}) - u(\mathbf{0})}{\overline{C}^{1-\sigma}} \approx \int_0^1 c_i di + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\int_0^1 c_i^2 di + \frac{1}{2}\left[(1-\sigma) - \left(\frac{\theta-1}{\theta}\right)\right]\left(\int_0^1 c_i di\right)^2.$$

Now consider the disutility of labor,

$$g(n) = \vartheta\frac{\overline{N}^{1+\psi}}{1+\psi}e^{(1+\psi)n}.$$

Note that in equilibrium,

$$\vartheta\overline{N}^{\psi}\overline{C}^{\sigma} = \overline{A} = \frac{\overline{C}}{\overline{N}} \iff \vartheta\overline{N}^{1+\psi} = \overline{C}^{1-\sigma}.$$

A quadratic approximation is given by,

$$\begin{aligned} g(n) &= \frac{\overline{C}^{1-\sigma}}{1+\psi}e^{(1+\psi)n} \\ &\approx \frac{\overline{C}^{1-\sigma}}{1+\psi}\left[1 + (1+\psi)\,n + \frac{1}{2}(1+\psi)^2 n^2\right], \end{aligned}$$

so that

$$
\begin{aligned}
\hat{U} &\equiv \frac{u(\mathbf{c}) - u(\mathbf{0})}{\overline{C}^{1-\sigma}} - \frac{g(n) - g(0)}{\overline{C}^{1-\sigma}} \\
&\approx \int_0^1 c_i di + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\int_0^1 c_i^2 di + \frac{1}{2}\left[(1-\sigma) - \left(\frac{\theta-1}{\theta}\right)\right]\left(\int_0^1 c_i di\right)^2 - n - \frac{1}{2}(1+\psi)n^2 \\
&= c - n - \frac{1}{2}\sigma c^2 - \frac{1}{2}\psi n^2 + \frac{1}{2}\int_0^1 c_i^2 di - \frac{1}{2}n^2 - \frac{1}{2\theta}\mathrm{Var}_i(c_i),
\end{aligned}
$$

where

$$
\mathrm{Var}_i(c_i) = \int_0^1 c_i^2 di - \left(\int_0^1 c_i di\right)^2
$$

Now consider the constraint terms associated with each of the problems.

$$
G_e \equiv \Lambda_e \left(E - \int_0^1 P_i C_i di\right).
$$

Let

$$
\begin{aligned}
\overline{\Lambda}_e &\equiv \frac{\overline{C}^{-\sigma}}{\overline{P}}, \\
\overline{E} &\equiv \overline{P} \times \overline{C}.
\end{aligned}
$$

Write

$$
\begin{aligned}
G_e &= \overline{\Lambda}_e e^{\lambda_e}\left(\overline{E}e^e - \overline{P} \times \overline{C}\int_0^1 e^{p_i + c_i} di\right) \\
&= \overline{C}^{1-\sigma} e^{\lambda_e}\left(e^e - \int_0^1 e^{p_i + c_i} di\right),
\end{aligned}
$$

so

$$
\frac{G_e}{\overline{C}^{1-\sigma}} = e^{\lambda_e}\left(e^e - \int_0^1 e^{p_i + c_i} di\right).
$$

Let

$$
f_e(\mathbf{c}, \lambda_e, \mathbf{p}, e) = e^{\lambda_e + e} - \int_0^1 e^{\lambda_e + p_i + c_i} di.
$$

Then

$$
\begin{aligned}
e^{\lambda_e + e} &\approx 1 + \lambda_e + e + \frac{1}{2}(\lambda_e + e)^2 \\
&= 1 + \lambda_e + e + \frac{1}{2}\lambda_e^2 + \lambda_e e + \frac{1}{2}e^2
\end{aligned}
$$

and

$$e^{(\lambda_e+p_i+c_i)} \approx 1 + (\lambda_e + p_i + c_i) + \frac{1}{2}(\lambda_e + p_i + c_i)^2$$
$$= 1 + (\lambda_e + p_i + c_i) + \frac{1}{2}\lambda_e^2 + \lambda_e (p_i + c_i) + \frac{1}{2}(p_i + c_i)^2$$

Therefore

$$\begin{aligned} f_e(\mathbf{c}, \lambda_e, \mathbf{p}, e) &\approx 1 + \lambda_e + e + \frac{1}{2}\lambda_e^2 + \lambda_e e + \frac{1}{2}e^2 \\ &\quad - \int_0^1 \left[1 + (\lambda_e + p_i + c_i) + \frac{1}{2}\lambda_e^2 + \lambda_e (p_i + c_i) + \frac{1}{2}(p_i + c_i)^2\right] di \\ &= e + \lambda_e e + \frac{1}{2}e^2 - \int_0^1 \left[(p_i + c_i) + \lambda_e (p_i + c_i) + \frac{1}{2}(p_i + c_i)^2\right] di \\ &= e + \frac{1}{2}e^2 - \int_0^1 (p_i + c_i)\, di + \lambda_e \left[e - \int_0^1 (p_i + c_i)\, di\right] - \frac{1}{2}\int_0^1 (p_i + c_i)^2 di \\ &= e + \frac{1}{2}e^2 - p - c + \lambda_e (e - p - c) - \frac{1}{2}\int_0^1 p_i^2 di - \int_0^1 p_i c_i di - \frac{1}{2}\int_0^1 c_i^2 di \end{aligned}$$

Now consider the constraint of step 2,

$$G_u = \Lambda_u \left(WN + \Pi - T - \int_0^1 P_i C_i di\right),$$

Analogously, we can write

$$G_u = \overline{\Lambda}_u e^{\lambda_u} \left(\overline{W} \times \overline{N} e^{w+n} + \overline{\Pi} e^{\ln\left(\frac{\Pi}{\overline{\Pi}}\right)} - \overline{T} e^{\ln\left(\frac{T}{\overline{T}}\right)} - \overline{P} \times \overline{C} \int_0^1 e^{p_i + c_i} di\right).$$

Note that $\overline{W} \times \overline{N} = \overline{P} \times \overline{C}$, and $\overline{\Pi} = \overline{T}$. Moreover, $\overline{\Lambda}_u = \overline{C}^{-\sigma}/\overline{P}$. Therefore

$$\frac{G_u}{\overline{C}^{1-\sigma}} = e^{\lambda_u + w + n} + \frac{\overline{\Pi}}{\overline{P}\,\overline{C}} \left[e^{\lambda_u + \ln\left(\frac{\Pi}{\overline{\Pi}}\right)} - e^{\lambda_u + \ln\left(\frac{T}{\overline{T}}\right)}\right] - \int_0^1 e^{\lambda_u + p_i + c_i} di$$

Now

$$\begin{aligned} \overline{\Pi} &= \left[\overline{P} - \left(\frac{\theta - 1}{\theta}\right)\frac{\overline{W}}{\overline{A}}\right]\overline{C} \\ &= \frac{1}{\theta}\overline{P}\;\overline{C}. \end{aligned}$$

Therefore

$$\begin{aligned}
\frac{G_u}{\overline{C}^{1-\sigma}} &= e^{\lambda_u+w+n} + \frac{1}{\theta}\left(e^{\lambda_u+\ln\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)} - e^{\lambda_u+\ln\left(\frac{T}{\overline{\overline{T}}}\right)}\right) - \int_0^1 e^{\lambda_u+p_i+c_i}di \\
&= w + \frac{1}{2}w^2 + n + \frac{1}{2}n^2 + wn + \lambda_u\left(w+n\right) + \\
&+ \frac{1}{\theta}\left[\ln\frac{\Pi}{\overline{\overline{\Pi}}} + \frac{1}{2}\ln^2\frac{\Pi}{\overline{\overline{\Pi}}} + \lambda_u\left(\ln\frac{\Pi}{\overline{\overline{\Pi}}} - \ln\frac{T}{\overline{\overline{T}}}\right) - \ln\frac{T}{\overline{\overline{T}}} - \frac{1}{2}\ln^2\frac{T}{\overline{\overline{T}}}\right] \\
&- \int_0^1\left[p_i + \frac{1}{2}p_i^2 + c_i + \frac{1}{2}c_i^2 + \lambda_u\left(p_i+c_i\right) + p_ic_i\right]di \\
&= n + \frac{1}{2}n^2 - c - \frac{1}{2}\int_0^1 c_i^2 di + wn - \int_0^1 p_ic_i di + \lambda_u\left[w + n + \frac{1}{\theta}\left(\ln\frac{\Pi}{\overline{\overline{\Pi}}} - \ln\frac{T}{\overline{\overline{T}}}\right) - p - c\right] \\
&+ w - p + \frac{1}{2}w^2 - \frac{1}{2}\int_0^1 p_i^2 di + \frac{1}{\theta}\left(\ln\frac{\Pi}{\overline{\overline{\Pi}}} + \frac{1}{2}\ln^2\frac{\Pi}{\overline{\overline{\Pi}}} - \ln\frac{T}{\overline{\overline{T}}} - \frac{1}{2}\ln^2\frac{T}{\overline{\overline{T}}}\right)
\end{aligned}$$

Summing utility with the constraint terms yield the results.

9.2. Proof of Lemma 3

Combining equation (19) and $\mathbb{E}\left[\Delta\hat{\mathcal{L}}_e \mid \mathbf{s}\right]$,

$$\begin{aligned}
\mathbb{E}\left[\Delta\hat{\mathcal{L}}_e \mid \mathbf{s}\right] &= -\frac{1}{2\theta}\mathbb{E}\left[\int_0^1\left[c - \int_0^1 \mu_{i|s}\left(p_i\right)di + \mu_{i|s}\left(p_i\right) - c_i^b\left(p_i\right)\right]^2 di \mid \mathbf{s}\right] \\
&= +\left(\frac{\theta\sigma-1}{2\theta}\right)\mathbb{E}\left[\left(\int_0^1\left[c + \mu_{i|s}\left(p_i\right) - \int_0^1 \mu_{i|s}\left(p_i\right)di - c_i^b\left(p_i\right)\right]di\right)^2 \mid \mathbf{s}\right].
\end{aligned}$$

Let $\int_0^1 \mu_{i|s}\left(p_i\right)di \equiv \overline{\mu}_{|s}$. The first expectation is

$$\mathbb{E}\left[\int_0^1\left[c - \overline{\mu}_{|s} + \mu_{i|s}\left(p_i\right) - c_i^b\left(p_i\right)\right]^2 di \mid \mathbf{s}\right] = \left(c - \overline{\mu}_{|s}\right)^2 + \int_0^1 \gamma_{i|s}^2\left(p_i\right)di.$$

The second expectation is

$$\begin{aligned}
\mathbb{E}\left[\left(\int_0^1\left[c - \overline{\mu}_{|s} + \mu_{i|s}\left(p_i\right) - c_i^b\left(p_i\right)\right]di\right)^2 \mid \mathbf{s}\right] &= \mathbb{E}\left[\left(c - \overline{\mu}_{|s} + \int_0^1\left[\mu_{i|s}\left(p_i\right) - c_i^b\left(p_i\right)\right]di\right)^2 \mid \mathbf{s}\right] \\
&= \left(c - \overline{\mu}_{|s}\right)^2 + \mathbb{E}\left[\left(\int_0^1\left[\mu_{i|s}\left(p_i\right) - c_i^b\left(p_i\right)\right]di\right)^2 \mid \mathbf{s}\right] \\
&= \left(c - \overline{\mu}_{|s}\right)^2.
\end{aligned}$$

The last equality results from the law of large numbers. Therefore

$$\mathbb{E}\left[\Delta\hat{\mathcal{L}}_e \mid \mathbf{s}\right] = -\frac{1}{2\theta}\int_0^1 \gamma_{i|s}^2 \left(p_i\right) di - \frac{1}{2}\sigma\left(c - \overline{\mu}_{|s}\right)^2.$$

Finally, letting $\overline{\mu} \equiv \int_0^1 \mu_i\left(p_i\right) di$,

$$\begin{aligned}\mathbb{E}\left[\left(c-\overline{\mu}_{|s}\right)^2\right] &= \mathbb{E}\left[\left(c-\overline{\mu}+\overline{\mu}-\overline{\mu}_{|s}\right)^2\right] \\ &= \left(c-\overline{\mu}\right)^2 + \mathbb{E}\left[\left(\overline{\mu}_{|s}-\overline{\mu}\right)^2\right] \\ &= \left(c-\overline{\mu}\right)^2,\end{aligned}$$

where the last equality follows again from the law of large numbers. Therefore

$$\mathbb{E}\left[\Delta\hat{\mathcal{L}}_e\right] = -\frac{1}{2\theta}\int_0^1 \gamma_{i|s}^2 \left(p_i\right) di - \frac{1}{2}\sigma\left(c-\overline{\mu}\right)^2,$$

which implies that only the first term depends on the distribution of the signal.

9.3. Proof of Lemma 4

Using the approximation $X = \overline{X}e^x$, we have

$$\Pi_i = \overline{P}\ \overline{C}\left(e^{p_i} - e^{w-a+\ln\left(\frac{1-\tau}{1-\overline{\tau}}\right)-\Theta}\right)e^{c-\theta(p_i-p)+\alpha\gamma_\epsilon\{\widetilde{\epsilon}_i-\mathbb{E}_i[\widetilde{\epsilon}]\}}.$$

Conditional on a price change, expected profits are

$$\Pi_i = \overline{P}\ \overline{C}\left(e^{p_i} - e^{w-a+\ln\left(\frac{1-\tau}{1-\overline{\tau}}\right)-\Theta}\right)e^{c-\theta(p_i-p)+\frac{1}{2}(\alpha\gamma_\epsilon)^2-\alpha\gamma_\epsilon\mathbb{E}_i[\widetilde{\epsilon}]}.$$

Taking the first-order condition with respect to p_i yields

$$p_i = w - a + \ln\left(\frac{1-\tau}{1-\overline{\tau}}\right) \equiv p_{\text{adj}}.$$

Therefore, optimized profits conditional on a price change are

$$\Pi_{\text{adj}} = \overline{P}\ \overline{C}\left(1-e^{-\Theta}\right)e^{-(\theta-1)\left(p_{\text{adj}}-p\right)+p+c-\alpha\gamma_\epsilon\mathbb{E}_i[\widetilde{\epsilon}]}e^{\frac{1}{2}(\alpha\gamma_\epsilon)^2}.$$

Conditional on keeping the price, profits are:

$$\Pi_{\text{no-adj}} = \overline{P}\,\overline{C}\left(1-e^{(p_{\text{adj}}-p)+\pi-\Theta}\right)e^{p+c+(\theta-1)\pi+\alpha\gamma_\epsilon(\epsilon_{i,0}-\mathbb{E}_i[\widetilde{\epsilon}])}.$$

Provided that

$$e^{(p_{\text{adj}}-p)+\pi-\Theta} < 1,$$

$\Pi_{\text{no-adj}}$ is strictly increasing in $\epsilon_{i,0}$, so a threshold rule is optimal. The threshold $\bar{\epsilon}$ is given by:

$$\Pi_{\text{no-adj}} = \Pi_{\text{adj}},$$

which implies:

$$\bar{\epsilon} = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left\{(\theta-1)\left[(p_{\text{adj}}-p)+\pi\right] + \ln\left[\frac{1-e^{(p_{\text{adj}}-p)+\pi-\Theta}}{1-e^{-\Theta}}\right]\right\}.$$

9.4. Proof of Proposition 2

We show the properties included in this proposition one at a time.

Uniqueness of $\bar{\epsilon}(\pi)$.. We first show that when $\pi < \Theta$, (37) is a well-defined function. Let

$$f(\bar{\epsilon},\pi) = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[(\theta-1)\frac{\pi}{\Phi(\bar{\epsilon})} + \ln\left(\frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}{1-e^{-\Theta}}\right)\right] - \bar{\epsilon}.$$

Note that $f(\bar{\epsilon},\pi)$ is only defined if $\pi < \Phi(\bar{\epsilon})\Theta$. Therefore, if $\pi < 0$, f is always well-defined. Otherwise, it is only defined for

$$\bar{\epsilon} > \Phi^{-1}\left(\frac{\pi}{\Theta}\right).$$

Hence, this function is only defined for $\pi < \Theta$. First suppose that $\pi < 0$. Then

$$\begin{aligned}
\lim_{\bar{\epsilon}\to-\infty} f(\bar{\epsilon},\pi) &= \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\ln\left(\frac{1}{1-e^{-\Theta}}\right)\right] - \lim_{\bar{\epsilon}\to-\infty}\left[\frac{1}{\alpha\gamma_\epsilon}(\theta-1)\frac{\pi}{\Phi(\bar{\epsilon})} - \bar{\epsilon}\right] \\
&= \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\ln\left(\frac{1}{1-e^{-\Theta}}\right)\right] - \lim_{\bar{\epsilon}\to-\infty}\left\{\frac{1}{\Phi(\bar{\epsilon})}\left[\frac{1}{\alpha\gamma_\epsilon}(\theta-1)\pi - \bar{\epsilon}\Phi(\bar{\epsilon})\right]\right\} \\
&= \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\ln\left(\frac{1}{1-e^{-\Theta}}\right)\right] - \frac{1}{\alpha\gamma_\epsilon}(\theta-1)\pi\lim_{\bar{\epsilon}\to-\infty}\left\{\frac{1}{\Phi(\bar{\epsilon})}\right\} = \infty,
\end{aligned}$$

and

$$\lim_{\bar{\epsilon}\to\infty} f(\bar{\epsilon},\pi) = \lim_{\bar{\epsilon}\to\infty}\left\{\frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[(\theta-1)\pi + \ln\left(\frac{1-e^{\pi-\Theta}}{1-e^{-\Theta}}\right)\right] - \bar{\epsilon}\right\} = -\infty.$$

The case where $\pi = 0$ is trivial, since in that case

$$\bar{\epsilon} = \frac{1}{2}\alpha\gamma_\epsilon.$$

When $\pi \in (0, \Theta)$,

$$\lim_{\overline{\epsilon} \to \Phi^{-1}\left(\frac{\pi}{\Theta}\right)} f(\overline{\epsilon}, \pi) = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[(\theta - 1)\frac{\pi}{\pi/\Theta} + \lim_{x \to 0} \ln(x)\right] - \Phi^{-1}\left(\frac{\pi}{\Theta}\right) = \infty.$$

Therefore, the equation $f(\overline{\epsilon}, \pi) = 0$ has at least one solution in $\overline{\epsilon}$. To show that it has only one solution, note that

$$\begin{aligned} f_{\overline{\epsilon}}(\overline{\epsilon}, \pi) &= -\frac{1}{\alpha\gamma_\epsilon}(\theta - 1)\left[-\frac{\pi}{\Phi^2(\overline{\epsilon})}\phi(\overline{\epsilon})\right] - \frac{1}{\alpha\gamma_\epsilon} \times \left[\frac{-e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}{1 - e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}\right] \times \left[-\frac{\pi}{\Phi^2(\overline{\epsilon})}\phi(\overline{\epsilon})\right] - 1 \\ &= \frac{1}{\alpha\gamma_\epsilon}\left[(\theta - 1) - \frac{e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}{1 - e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}\right]\left[\frac{\pi}{\Phi^2(\overline{\epsilon})}\phi(\overline{\epsilon})\right] - 1. \end{aligned}$$

Note that

$$\begin{aligned} \ln\left(\frac{\theta}{\theta - 1}\right) &= \Theta \\ \Longleftrightarrow \theta - 1 &= \frac{e^{-\Theta}}{1 - e^{-\Theta}}. \end{aligned}$$

Therefore

$$f_{\overline{\epsilon}}(\overline{\epsilon}, \pi) = \frac{1}{\alpha\gamma_\epsilon}\left[\frac{1 - e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}{1 - e^{-\Theta}} - e^{\frac{\pi}{\Phi(\overline{\epsilon})}}\right]\frac{e^{-\Theta}}{1 - e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}\frac{\pi}{\Phi(\overline{\epsilon})}\left[\frac{\phi(\overline{\epsilon})}{\Phi(\overline{\epsilon})}\right] - 1.$$

It is easy to show that the first term in square brackets is negative as long as

$$0 < \frac{\pi}{\Phi(\overline{\epsilon})}.$$

Therefore the first term in $f_{\overline{\epsilon}}(\overline{\epsilon}, \pi)$ is negative, which implies that $f(\overline{\epsilon}, \pi)$ is strictly decreasing in $\overline{\epsilon}$. Hence, there is a unique solution for $f(\overline{\epsilon}, \pi) = 0$, and the implicit function theorem globally defined $\overline{\epsilon}(\pi)$. Now

$$\begin{aligned} f_\pi(\overline{\epsilon}, \pi) &= -\frac{1}{\alpha\gamma_\epsilon}\frac{e^{-\Theta}}{1 - e^{-\Theta}}\frac{1}{\Phi(\overline{\epsilon})} - \frac{1}{\alpha\gamma_\epsilon}\frac{-\frac{1}{\Phi(\overline{\epsilon})}e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}{1 - e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}} \\ &= \frac{1}{\alpha\gamma_\epsilon}\frac{1}{\Phi(\overline{\epsilon})}\frac{e^{-\Theta}}{1 - e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}\left[e^{\frac{\pi}{\Phi(\overline{\epsilon})}} - \frac{1 - e^{\frac{\pi}{\Phi(\overline{\epsilon})} - \Theta}}{1 - e^{-\Theta}}\right]. \end{aligned}$$

The implicit function theorem then yields

$$\bar{\epsilon}'(\pi) = -\frac{f_\pi(\bar{\epsilon}(\pi),\pi)}{f_{\bar{\epsilon}}(\bar{\epsilon}(\pi),\pi)}$$
$$= \frac{\frac{1}{\alpha\gamma_\epsilon}\frac{1}{\Phi(\bar{\epsilon})}\frac{e^{-\Theta}}{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}\left[e^{\frac{\pi}{\Phi(\bar{\epsilon})}} - \frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}{1-e^{-\Theta}}\right]}{1+\frac{1}{\alpha\gamma_\epsilon}\left[e^{\frac{\pi}{\Phi(\bar{\epsilon})}} - \frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}{1-e^{-\Theta}}\right]\frac{e^{-\Theta}}{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}\frac{\pi}{\Phi(\bar{\epsilon})}\left[\frac{\phi(\bar{\epsilon})}{\Phi(\bar{\epsilon})}\right]},$$

or

$$\bar{\epsilon}'(\pi)\,\pi = \frac{\Omega(\pi)}{1+\frac{\phi[\bar{\epsilon}(\pi)]}{\Phi[\bar{\epsilon}(\pi)]}\Omega(\pi)},$$

where

$$\Omega(\pi) \equiv \frac{1}{\alpha\gamma_\epsilon}\left[\frac{e^{\varphi(\pi)-\Theta}}{1-e^{\varphi(\pi)-\Theta}} - \frac{e^{-\Theta}}{1-e^{-\Theta}}\right]\varphi(\pi),$$

and $\varphi(\pi) \equiv \pi/\Phi[\bar{\epsilon}(\pi)]$.

Minimum at $\pi = 0$.. Note that $\bar{\epsilon}'(\pi) > 0 \iff \pi > 0$. Therefore $\bar{\epsilon}(\pi)$ has a minimum at $\pi = 0$. Therefore, $\chi \equiv 1 - \Phi(\bar{\epsilon})$, the fraction of sticky firms, has a maximum at $\pi = 0$.

Limits as $\pi \to \Theta$ or $\pi \to -\infty$.. When $\pi \to \Theta$, we have

$$\lim_{\pi\to\Theta}\bar{\epsilon}(\pi) = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left\{\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{\Theta}{\lim_{\pi\to\Theta}\Phi(\bar{\epsilon}(\pi))} + \ln\left[\frac{1-e^{\frac{\Theta}{\lim_{\pi\to\Theta}\Phi(\bar{\epsilon}(\pi))}-\Theta}}{1-e^{-\Theta}}\right]\right\}.$$

The term in the logarithm is smaller than zero unless $\Phi[\bar{\epsilon}(\pi)] \to 1$, i.e., $\bar{\epsilon}(\pi) \to \infty$. Moreover, $\bar{\epsilon}(\pi) \to \infty$ is a fixed-point of the equation above and, for all $\pi < \Theta$, there is a unique $\bar{\epsilon}(\pi)$. Therefore $\lim_{\pi\to\Theta}\bar{\epsilon}(\pi) = \infty$. For $\pi \to -\infty$, note that $\Phi[\bar{\epsilon}(\pi)] \in [0,1]$ implies that

$$\ln\left[\frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon}(\pi))}-\Theta}}{1-e^{-\Theta}}\right] \to \ln\left[\frac{1}{1-e^{-\Theta}}\right],$$

and

$$\frac{\pi}{\Phi[\bar{\epsilon}(\pi)]} \to -\infty.$$

These two facts imply that $\bar{\epsilon}(\pi) \to \infty$ (and that therefore $\Phi[\bar{\epsilon}(\pi)] \to 1$).

Asymmetry of $\overline{\epsilon}(\pi)$.. To derive the asymmetry in $\overline{\epsilon}(\pi)$, it is convenient to write

$$\overline{\epsilon}(\pi) = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{\pi}{\Phi(\overline{\epsilon})} + \ln\left(\frac{1-e^{\frac{\pi}{\Phi(\overline{\epsilon})}-\Theta}}{1-e^{-\Theta}}\right)\right].$$

Let a be some strictly positive scalar. Clearly, if $a \geq \Theta$, $\overline{\epsilon}(a) = \infty > \overline{\epsilon}(-a)$. Now consider $a \in (0, \Theta)$. Since $f(\overline{\epsilon}, \pi)$ is strictly decreasing in $\overline{\epsilon}$, to show that $\overline{\epsilon}(a) > \overline{\epsilon}(-a)$ it suffices to show that $f[\overline{\epsilon}(-a), a] > 0$. Now

$$\overline{\epsilon}(-a) = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{-a}{\Phi[\overline{\epsilon}(-a)]} + \ln\left(\frac{1-e^{\frac{-a}{\Phi[\overline{\epsilon}(-a)]}-\Theta}}{1-e^{-\Theta}}\right)\right],$$

and

$$f[\overline{\epsilon}(-a), a] = \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{a}{\Phi[\overline{\epsilon}(-a)]} + \ln\left(\frac{1-e^{\frac{a}{\Phi[\overline{\epsilon}(-a)]}-\Theta}}{1-e^{-\Theta}}\right)\right] - \overline{\epsilon}(-a).$$

Therefore

$$\begin{aligned} f[\overline{\epsilon}(-a), a] &= \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{a}{\Phi[\overline{\epsilon}(-a)]} + \ln\left(\frac{1-e^{\frac{a}{\Phi[\overline{\epsilon}(-a)]}-\Theta}}{1-e^{-\Theta}}\right)\right] - \overline{\epsilon}(-a) \\ &\propto \ln\left(\frac{1-e^{\frac{-a}{\Phi[\overline{\epsilon}(-a)]}-\Theta}}{1-e^{\frac{a}{\Phi[\overline{\epsilon}(-a)]}-\Theta}}\right) - 2\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{a}{\Phi[\overline{\epsilon}(-a)]}. \end{aligned}$$

Let

$$\omega \equiv \frac{a}{\Phi[\overline{\epsilon}(-a)]},$$

and consider the function

$$g(\omega) \equiv \ln\left(\frac{1-e^{-\omega-\Theta}}{1-e^{\omega-\Theta}}\right) - 2\frac{e^{-\Theta}}{1-e^{-\Theta}}\omega.$$

Clearly $g(0) = 0$. Moreover,

$$g'(\omega) = \frac{1}{e^{\Theta+\omega}-1} + \frac{1}{e^{\Theta-\omega}-1} - \frac{2}{e^{\Theta}-1}.$$

Again, $g'(0) = 0$, and

$$g''(\omega) = \frac{e^{\Theta-\omega}}{(e^{\Theta-\omega}-1)^2} - \frac{e^{\Theta+\omega}}{(e^{\Theta+\omega}-1)^2}.$$

Now note that

$$\begin{aligned}
h(t) &= \frac{e^t}{(e^t-1)^2} \\
\Longrightarrow h'(t) &= \frac{e^t(e^t-1) - e^t \times 2(e^t-1)e^t}{(e^t-1)^4} \\
\Longrightarrow h'(t) &\propto -\left(1+e^t\right) < 0,
\end{aligned}$$

for $t > 0$. Therefore, for $\omega > 0$,

$$g''(\omega) > 0 \Longrightarrow g'(\omega) > 0 \Longrightarrow g(\omega) > 0,$$

which implies that $f[\bar{\epsilon}(-a), a] > 0$ for $a > 0$.

9.5. Proof of Proposition 1 (existence of equilibrium)

Define $\mathcal{E}(\pi) = \pi + \widetilde{c}(\pi)$. Observe that $\widetilde{c}(\pi) \to 0$ as $\pi \to \pm\infty$. Therefore,

$$\lim_{\pi\to-\infty} \mathcal{E}(\pi) = -\infty, \quad \lim_{\pi\to\infty} \mathcal{E}(\pi) = \infty,$$

Implying that the equation $\mathcal{E}(\pi) = -c^*$ has at least one solution by the intermediate value theorem.

To show uniqueness, consider the derivative:

$$\begin{aligned}
\widetilde{c}'(\pi) &= \frac{1}{\psi+\sigma}\left\{\left[\frac{\chi'(\pi)}{[1-\chi(\pi)]^2}\right]\pi + \frac{\chi(\pi)}{1-\chi(\pi)}\right\} \\
&= \frac{1}{\psi+\sigma}\left[\frac{1}{1-\chi(\pi)}\right]\left\{\chi(\pi) - \frac{\frac{\phi[\bar{\epsilon}(\pi)]}{\Phi[\bar{\epsilon}(\pi)]}\Omega(\pi)}{1+\frac{\phi[\bar{\epsilon}(\pi)]}{\Phi[\bar{\epsilon}(\pi)]}\Omega(\pi)}\right\} \\
&> \frac{1}{\psi+\sigma}\left[\frac{1}{1-\chi(\pi)}\right]\{\chi(\pi)-1\} \\
&= -\frac{1}{\psi+\sigma}.
\end{aligned}$$

Hence, $\mathcal{E}'(\pi) = 1 + \widetilde{c}'(\pi) > 1 - \frac{1}{\psi+\sigma} \geq 0$, which implies that $\mathcal{E}(\pi)$ is strictly increasing and ensures uniqueness when $\psi + \sigma \geq 1$. Therefore,

$$\mathcal{E}'(\pi) = 1 + \widetilde{c}'(\pi) > 1 - \frac{1}{\psi+\sigma}.$$

If $\psi + \sigma \geq 1$, it follows that $\mathcal{E}'(\pi) > 0$ for all π. Hence, $\mathcal{E}(\pi)$ is strictly increasing, and the equilibrium is unique.

9.6. Proof of Proposition 3 ("rockets and feathers")

The equilibrium condition can be written as,

$$\mathcal{E}(\pi) \equiv \pi + \widetilde{c}(\pi) = -c^*.$$

Assume $m = 0$. The equilibrium condition for inflation is

$$\pi + \widetilde{c}(\pi) = -\nu.$$

To show that $\pi(\nu)\nu < 0$, consider the function

$$\mathcal{E}(\pi) = \pi + \widetilde{c}(\pi) = \left\{1 + \frac{1}{\psi + \sigma}\left[\frac{\chi(\pi)}{1 - \chi(\pi)}\right]\right\}\pi.$$

The expression in curly brackets is strictly positive, so $\text{sign}(\pi(\nu)) = -\text{sign}(\nu)$.

Since $\mathcal{E}'(\pi) > 0$, it suffices to show that

$$\mathcal{E}[-\pi(\nu)] < \nu.$$

Note that

$$\mathcal{E}[-\pi(\nu)] = -\pi(\nu) + \widetilde{c}[-\pi(\nu)],$$

and from the equilibrium condition,

$$\pi(\nu) + \widetilde{c}[\pi(\nu)] = -\nu.$$

Therefore,

$$\begin{aligned}\mathcal{E}[-\pi(\nu)] < \nu &\iff -\pi(\nu) + \widetilde{c}[-\pi(\nu)] < \nu \\ &\iff -\left[-\nu - \widetilde{c}[\pi(\nu)]\right] + \widetilde{c}[-\pi(\nu)] < \nu \\ &\iff \widetilde{c}[\pi(\nu)] + \widetilde{c}[-\pi(\nu)] < 0.\end{aligned}$$

Substituting the expression for $\widetilde{c}(\pi)$, we obtain:

$$\begin{aligned}&\frac{1}{\psi + \sigma}\left[\frac{\chi(\pi(\nu))}{1 - \chi(\pi(\nu))}\right]\pi(\nu) + \frac{1}{\psi + \sigma}\left[\frac{\chi(-\pi(\nu))}{1 - \chi(-\pi(\nu))}\right](-\pi(\nu)) < 0 \\ \iff &\left[\frac{\chi(\pi(\nu))}{1 - \chi(\pi(\nu))} - \frac{\chi(-\pi(\nu))}{1 - \chi(-\pi(\nu))}\right]\pi(\nu) < 0.\end{aligned}$$

Since $\pi(\nu) < 0$, the inequality above holds if and only if

$$\frac{\chi(\pi(\nu))}{1 - \chi(\pi(\nu))} > \frac{\chi(-\pi(\nu))}{1 - \chi(-\pi(\nu))},$$

which is equivalent to

$$\chi(\pi(\nu)) > \chi(-\pi(\nu)).$$

This inequality follows directly from Proposition 2, completing the proof.

9.7. Proof of Lemma 5

We first derive the expression for $\Delta(\pi)$. From equation (25),

$$\mathrm{Var}_i[c_i] = \theta^2 \mathrm{Var}(p_i - p) + (\alpha\gamma_\epsilon)^2 \mathrm{Var}[\widetilde{\epsilon}_i - \mathbb{E}_i[\widetilde{\epsilon}_i]] - 2\theta\alpha\gamma_\epsilon \mathrm{Cov}[p_i - p, \widetilde{\epsilon}_i - \mathbb{E}_i[\widetilde{\epsilon}_i]].$$

First note that

$$\mathrm{Var}_i(p_i - p) = \frac{\chi}{1-\chi}\pi^2.$$

Second,

$$\mathbb{E}_i[\widetilde{\epsilon}_i] = \chi \mathbb{E}[\epsilon_{i,0} \mid \epsilon_{i,0} \geq \widetilde{\epsilon}] = \phi(\overline{\epsilon}).$$

Moreover, from the properties of the truncated normal distribution,

$$\mathbb{E}_i\left[\widetilde{\epsilon}_i^2\right] = \chi\left[1 + \frac{\overline{\epsilon}\phi(\overline{\epsilon})}{\chi}\right] + (1-\chi) = 1 + \overline{\epsilon}\phi(\overline{\epsilon}),$$

from which it follows that

$$\mathrm{Var}_i[\widetilde{\epsilon}_i - \mathbb{E}_i[\widetilde{\epsilon}_i]] = \mathbb{E}_i\left[\widetilde{\epsilon}_i^2\right] - (\mathbb{E}_i[\widetilde{\epsilon}_i])^2 = 1 + \overline{\epsilon}\phi(\overline{\epsilon}) - [\phi(\overline{\epsilon})]^2.$$

As for the covariance, note that

$$\begin{aligned}
\mathrm{Cov}_i[p_i - p, \widetilde{\epsilon}_i - \mathbb{E}_i[\widetilde{\epsilon}_i]] &= \mathbb{E}_i[(p_i - p)(\widetilde{\epsilon}_i - \mathbb{E}_i[\widetilde{\epsilon}_i])] \\
&= \chi \times (-\pi)\left[\frac{\phi(\overline{\epsilon})}{\chi} - \phi(\overline{\epsilon})\right] + (1-\chi)(p_{\mathrm{adj}} - p)[-\phi(\overline{\epsilon})] \\
&= -\pi\phi(\overline{\epsilon})
\end{aligned}$$

Therefore

$$\mathrm{Var}_i[c_i] = \theta^2\left(\frac{\chi}{1-\chi}\right)\pi^2 + \alpha^2\gamma_\epsilon^2\left[1 + \overline{\epsilon}\phi(\overline{\epsilon}) - \phi(\overline{\epsilon})^2\right] + 2\theta\alpha\gamma_\epsilon\pi\phi(\overline{\epsilon}).$$

As for cognitive costs,

$$\mathcal{I} = \frac{\kappa}{2}\left[\chi \times 0 + (1-\chi)\left(\ln\gamma_c^2 - \ln\theta\kappa\right)\right] = \frac{1}{2\theta}\Phi[\overline{\epsilon}(\pi)]\theta\kappa\ln\left(\frac{\gamma_c^2}{\theta\kappa}\right).$$

Now,

$$\alpha = 1 - \frac{\theta\kappa}{\gamma_c^2},$$

and

$$\gamma_\epsilon = \sqrt{\frac{\theta\kappa}{\alpha}} \iff \theta\kappa = \alpha\gamma_\epsilon^2.$$

Therefore

$$\alpha = 1 - \frac{\theta\kappa}{\gamma_c^2} \iff \frac{\theta\kappa}{\gamma_c^2} = 1 - \alpha,$$

from which it follows that

$$\mathcal{I} = \frac{1}{2\theta}\Phi\left[\bar{\epsilon}\left(\pi\right)\right]\alpha\gamma_\epsilon^2 \ln\left(\frac{1}{1-\alpha}\right).$$

We start by showing that the implementable set of equilibria is characterized by equation (35).

Recall that the equilibrium conditions for c, n, π, and $\bar{\epsilon}$ can be summarized as

$$\sigma c + \psi n = \frac{1-\Phi\left(\bar{\epsilon}\right)}{\Phi\left(\bar{\epsilon}\right)}\pi - \ln\left(\frac{1-\tau}{1-\overline{\tau}}\right) + a, \tag{41}$$

$$c = a + n, \tag{42}$$

$$c + \pi = m, \tag{43}$$

$$\bar{\epsilon} = \begin{cases} \frac{1}{2}\alpha\gamma_\epsilon - \frac{1}{\alpha\gamma_\epsilon}\left[\frac{e^{-\Theta}}{1-e^{-\Theta}}\frac{\pi}{\Phi(\bar{\epsilon})} + \ln\left(\frac{1-e^{\frac{\pi}{\Phi(\bar{\epsilon})}-\Theta}}{1-e^{-\Theta}}\right)\right], & \text{if } \pi < \Theta \\ \infty, & \text{if } \pi \geq \Theta \end{cases}. \tag{44}$$

Given π, equation (44) determines $\bar{\epsilon}$. Given c and n, equation (43) determines m, while equation (41) determines τ.

Since the set of implementable equilibria is characterized by equation (35), we can write the (non-linear) Lagrangian associated with the Ramsey problem as

$$\mathcal{L}_p = U + \Lambda_p\left(AN - \int_0^1 C_i di\right).$$

From Lemma 1,

$$\hat{U} = c - \frac{1}{2}\sigma c^2 + \frac{1}{2}\int_0^1 c_i^2 di - n - \frac{1+\psi}{2}n^2 - \frac{1}{2\theta}\text{Var}_i\left[c_i\right]$$

To derive the second-order approximation of the constraint term, write

$$\begin{aligned} G_p &= \Lambda_p\left(AN - \int_0^1 C_i di\right) \\ &= \overline{\Lambda}_p e^{\lambda_p}\left(\overline{A}\times\overline{N}e^{a+n} - \overline{C}\int_0^1 e^{c_i}di\right). \end{aligned}$$

So, again, we can write

$$
\begin{aligned}
\frac{G_p}{\overline{C}^{1-\sigma}} &= e^{\lambda_p+a+n} - \int_0^1 e^{\lambda_p+c_i} di \\
&\approx 1 + \lambda_p + a + n + \frac{1}{2}\left(\lambda_p + a + n\right)^2 - \\
&- \int_0^1 \left(1 + \lambda_p + c_i\right) di - \frac{1}{2}\int_0^1 \left(\lambda_p + c_i\right)^2 di \\
&= a + n + \frac{1}{2}\left(\lambda_p + a + n\right)^2 - \int_0^1 c_i di - \frac{1}{2}\int_0^1 \left(\lambda_p + c_i\right)^2 di \\
&= a + n + \lambda_p\left(a + n\right) + \frac{1}{2}\left(a + n\right)^2 - \int_0^1 c_i di - \lambda c - \frac{1}{2}\int_0^1 c_i^2 di \\
&= a + \frac{1}{2}a^2 + n + \frac{1}{2}n^2 + \lambda_p\left(a + n - c\right) + an - c - \frac{1}{2}\int_0^1 c_i^2 di
\end{aligned}
$$

Letting

$$\hat{\mathcal{L}}_p = \hat{U} - \mathcal{I} + \frac{G_p}{\overline{C}^{1-\sigma}}$$

yields the result. The solution to c and n follows from taking first-order conditions with respect to c, n, and λ_p.

9.8. Proof of Lemma 6

It is easy to show that

$$\Delta\left(\Theta\right) = \alpha^2\gamma_\epsilon^2 + \alpha\gamma_\epsilon^2 \ln\left(\frac{1}{1-\alpha}\right),$$

and

$$\Delta\left(0\right) = \alpha^2\gamma_\epsilon^2\left\{1 + \bar{\epsilon}\left(0\right)\phi\left[\bar{\epsilon}\left(0\right)\right] - \phi^2\left[\bar{\epsilon}\left(0\right)\right]\right\} + \Phi\left[\bar{\epsilon}\left(0\right)\right]\alpha\gamma_\epsilon^2 \ln\left(\frac{1}{1-\alpha}\right).$$

Therefore

$$
\begin{aligned}
&\Delta\left(\Theta\right) - \Delta\left(0\right) = \left\{1 - \Phi\left[\bar{\epsilon}\left(0\right)\right]\right\}\alpha\gamma_\epsilon^2 \ln\left(\frac{1}{1-\alpha}\right) - \alpha^2\gamma_\epsilon^2\left\{\bar{\epsilon}\left(0\right)\phi\left[\bar{\epsilon}\left(0\right)\right] - \phi^2\left[\bar{\epsilon}\left(0\right)\right]\right\} \\
\iff &\frac{\Delta\left(\Theta\right) - \Delta\left(0\right)}{\alpha\gamma_\epsilon^2\phi\left[\bar{\epsilon}\left(0\right)\right]} = \left[\frac{1 - \Phi\left(\frac{1}{2}\sqrt{\alpha\theta\kappa}\right)}{\phi\left(\frac{1}{2}\sqrt{\alpha\theta\kappa}\right)}\right]\ln\left(\frac{1}{1-\alpha}\right) - \alpha\left[\frac{1}{2}\sqrt{\alpha\theta\kappa} - \phi\left(\frac{1}{2}\sqrt{\alpha\theta\kappa}\right)\right].
\end{aligned}
$$

As $\alpha \to 0$, the expression goes to zero. As $\alpha \to 1$, the expression above goes to ∞. Therefore, there must be a $\overline{\alpha}$ (potentially zero) such that if $\alpha \geq \overline{\alpha}$, then $\Delta(\Theta) - \Delta(0) > 0$. But

$$\alpha = 1 - \frac{\theta\kappa}{\gamma_c^2}.$$

Therefore

$$\alpha \geq \overline{\alpha} \iff \gamma_c^2 \geq \frac{\theta\kappa}{1-\overline{\alpha}} \equiv \overline{\gamma}_c^2.$$

9.9. Proof of Lemma 8

Using the logarithmic approximation,

$$\begin{aligned}
\overline{\Pi}e^{r_t} &= \left(\overline{P}e^{p_t} - \overline{\Xi}e^{\xi_t}\right)\overline{P}^{-\theta}e^{-\theta p_t}\mathbb{E}\left(e^{\alpha\gamma_\epsilon\widetilde{\epsilon}_t}\right) \\
\iff \frac{1}{\theta}e^{r_t} &= \left(e^{p_t} - \frac{\theta-1}{\theta}e^{\xi_t}\right)e^{-\theta p_t}\mathbb{E}\left(e^{\alpha\gamma_\epsilon\widetilde{\epsilon}_t}\right) \\
\iff e^{r_t} &= \left[\theta e^{p_t} - (\theta-1)e^{\xi_t}\right]e^{-\theta p_t}\mathbb{E}\left(e^{\alpha\gamma_\epsilon\widetilde{\epsilon}_t}\right) \\
\iff r_t &= \ln\left[\theta e^{p_t} - (\theta-1)e^{\xi_t}\right] - \theta p_t + \ln\left[\mathbb{E}\left(e^{\alpha\gamma_\epsilon\widetilde{\epsilon}_t}\right)\right]
\end{aligned}$$

The standard second-order approximation of $\ln\left[\theta e^{p_t} - (\theta-1)e^{\xi_t}\right]$ around $p_t = 0$ and $\xi_t = 0$ yields

$$\ln\left[\theta e^{p_t} - (\theta-1)e^{\xi_t}\right] \approx \theta p_t - (\theta-1)\xi_t - \frac{\theta(\theta-1)}{2}(p_t - \xi_t)^2.$$

Moreover,

$$\ln\left(\mathbb{E}\left[e^{\alpha\gamma_\epsilon\widetilde{\epsilon}_t}\right]\right) = \begin{cases} \frac{1}{2}(\alpha\gamma_\epsilon)^2, & \text{if } p_t \neq p_{t-1} \\ \alpha\gamma_\epsilon\epsilon_{t-1}, & \text{if } p_t = p_{t-1} \end{cases}.$$

Plugging into r_t yields the result.